AMERICAN BANKRUPTCY INSTITUTE

2021 Virtual Annual Spring Meeting

The State of the Industry: Perspectives, Opportunities and Predictions

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PLENARY SESSION

2021

The State of the Restructuring Industry - Perspectives, Plans, and "Predictions"

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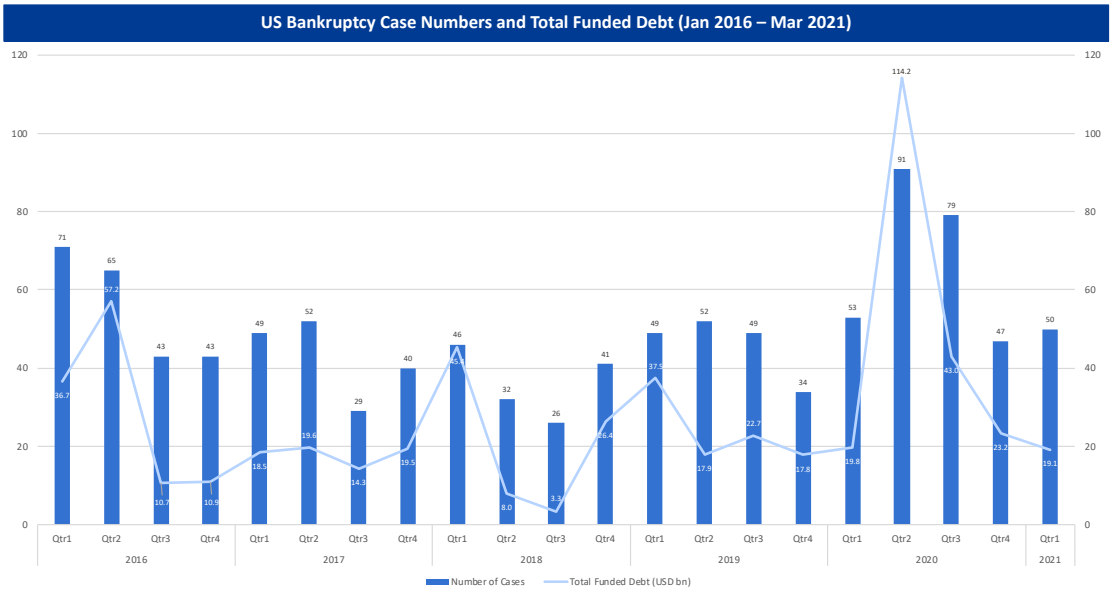

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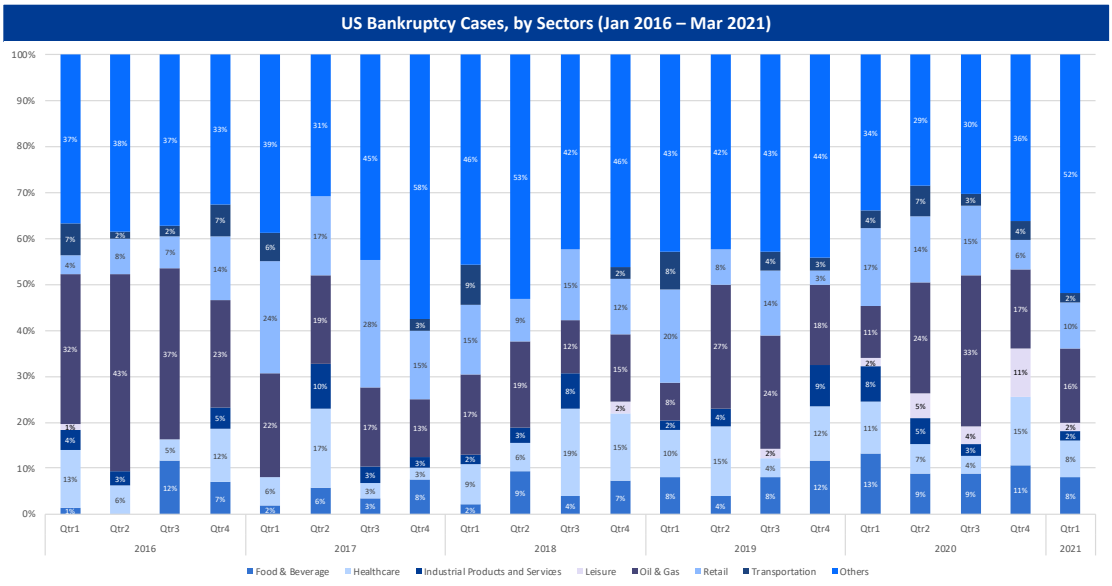

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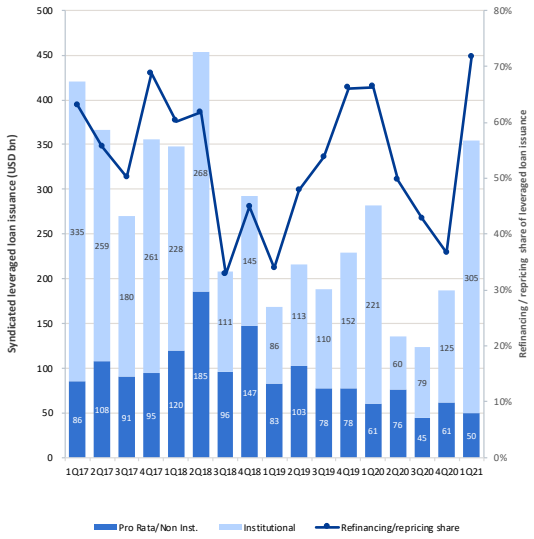
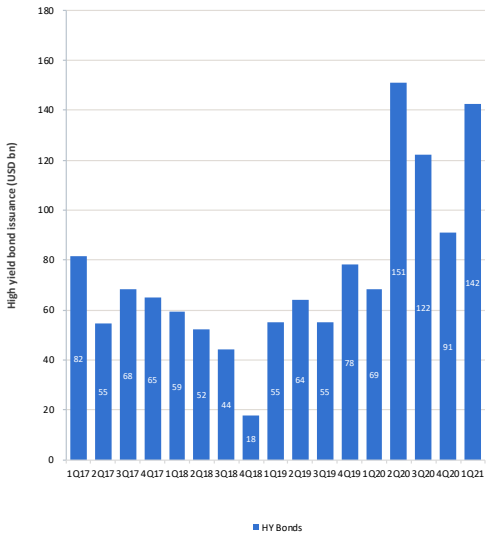
Borrowers tap the abundant liquidity in the leveraged loan & HY bond markets





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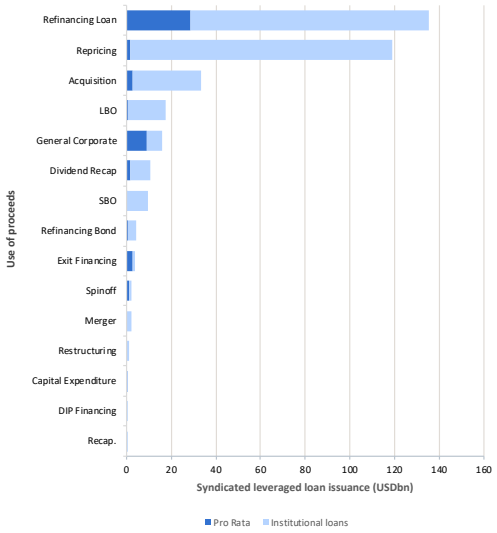
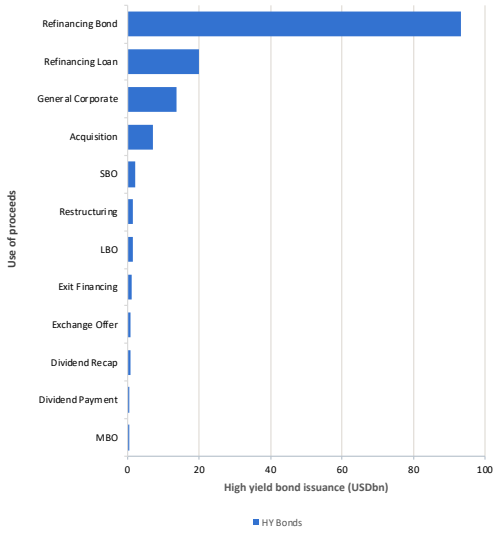
Repricings & refinancings drive loan volume in 1Q21.....



.....while refinancings lead bond activity



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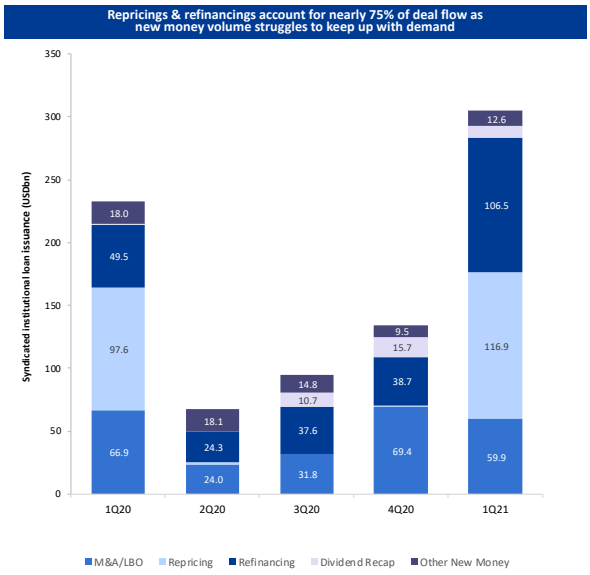

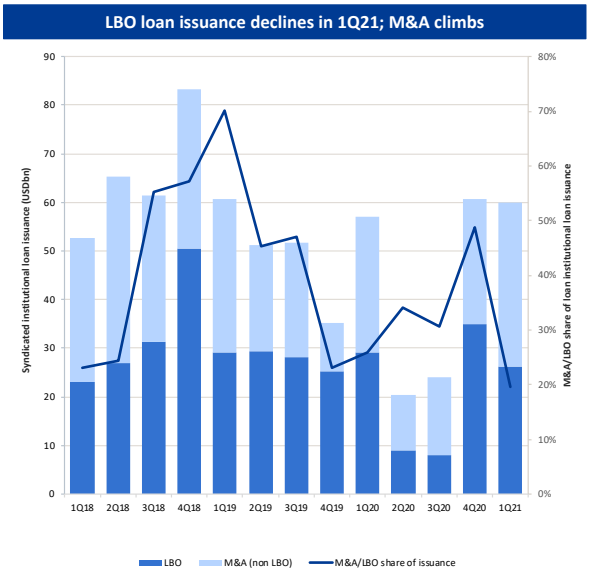

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Paycheck Protection Program

Where Are We Now? An Up-to-Date Guide to the Paycheck Protection Program

Coronavirus Resource Center

Proskauer's cross-disciplinary, cross-jurisdictional Coronavirus Response Team is focused on supporting and addressing client concerns. We will continue to evaluate the CARES Act, the Consolidated Appropriations Act, 2021, related regulations and any subsequent legislation to provide our clients guidance in real time. Please visit our Coronavirus Resource Center for guidance on risk management measures, practical steps businesses can take, and resources to help manage ongoing operations.

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PAYCHECK PROTECTION PROGRAM – WHERE ARE WE NOW?

An up-to-date guide to the Paycheck Protection Program

Last updated as of April 8, 2021

Since the enactment of the Coronavirus Aid, Relief, and Economic Security Act (the "CARES Act") on March 27, 2020, Congress has enacted four subsequent laws and the U.S. Small Business Administration (the "SBA") and the U.S Treasury Department ("Treasury") have issued a sizable number of rules and additional guidance to implement the CARES Act's marquee small business loan component – the Paycheck Protection Program (the "PPP").

Under the CARES Act, as supplemented by companion legislation such as the Paycheck Protection Program and Health Care Enhancement Act (the "PPPHCEA")[i], HR 7010 (Paycheck Protection Program Flexibility Act of 2020 (the "PPPFA")) and as extended under S.4116, the total amount available for emergency lending under this unprecedented program reached $659 billion. By the end of the initial PPP availability period in August 2020, approximately 5.2 million PPP loans had been issued by thousands of financial institutions to small businesses for an aggregate principal amount of approximately $522 billion.

On December 27, 2020, the Consolidated Appropriations Act, 2021 (the "CAA"), which provides $900 billion in new COVID-19 relief funding, was signed into law. Title III of the CAA, the Economic Aid to Hard-Hit Small Businesses, Nonprofits and Venues Act (the "Economic Aid Act"), expands upon the PPP implemented by the CARES Act and related legislation. The Economic Aid Act appropriates a total of $284.45 billion for the PPP, including the new Paycheck Protection Program Second Draw Loan Program (the "Second Draw Program" and loans made under such program, "Second Draw Loans")[ii], composed of $147.45 billion of additional funding for the existing PPP and $137 billion for the Second Draw Program. This additional funding for the existing PPP brought total appropriations for that program to $806.45 billion.

Since the beginning of the program the SBA and Treasury have issued a number of Interim Final Rules and SBA Procedural Notices governing the PPP (collectively, generally referred to herein as the "PPP Rules").[iii] The SBA and Treasury have also published: a borrower application form for all Schedule C (or F) filers using gross income (SBA Form 2483-C), a borrower application form for all other types of entities (SBA Form 2483), and lender application form (SBA Form 2484) for First Draw PPP loans; a borrower application form for all Schedule C (or F) filers using gross income (SBA Form 2483-SD-C), a borrower application form for all other types of entities (SBA Form 2483-SD), and lender application form (SBA Form 2484-SD) for Second Draw Loans; program "fact sheets" for borrowers and lenders; a summary of the applicable affiliation rules; a forgiveness application form (SBA Form 3508), simplified forgiveness application (SBA Form 3508EZ) for certain borrowers and a short forgiveness application form for borrowers of $150,000 or less (SBA Form 3508S); and responses to certain Frequently Asked Questions (the "FAQ") (which the SBA has updated numerous times) and Frequently Asked Questions on Loan Forgiveness (the "Loan Forgiveness FAQ"). In late 2020 the SBA published necessity questionnaires for each of for-profit borrowers (SBA Form 3509) and non-profit borrowers (SBA Form 3510) that must be completed and submitted by each PPP borrower that together with its

affiliates received PPP loans with an original principal amount of $2 million or greater within ten business days of receipt of such amount from such lender.

On January 6, 2021, the SBA published two new Interim Final Rules: the first addresses the PPP and loans thereunder (referred to herein as "PPP loans" or "First Draw PPP loans") as amended by the Economic Aid Act and amends, consolidates, and restates in a single document the rules governing borrower eligibility, lender eligibility, and loan application and origination requirements, as well as general rules on increases and loan forgiveness for PPP loans (the "EAA Updated Rules"); and the second provides rules implementing the Second Draw Program (the "Second Draw Rules"). The SBA also issued a consolidated interim final rule on January 19, 2021 governing all aspects of loan forgiveness and loan review. The latest interim final rule was issued by the SBA on March 18, 2021.

On March 11, 2021, the American Rescue Plan Act (the "ARPA") was signed into law, which modifies the SBA affiliation rules for 501(c)(3) organizations, such that 501(c)(3) organizations that do not employ more than 500 employees **per physical location**, rather than together with its affiliates, will become eligible to receive loans. Moreover, an additional $7.25 billion will be provided for the program. This additional funding for the existing PPP brings total appropriations for that program to $813.7 billion.

On March 30, 2021, the PPP Extension Act, which extends the Paycheck Protection Program until May 31, 2021, was signed into law. The PPP Extension Act gives applicants two additional months to apply for a First Draw or Second Draw PPP loan and gives the SBA until June 30, 2021 to process loan applications.

This alert (I) summarizes the key terms of the PPP (as amended and supplemented by the Economic Aid Act, the EAA Updated Rules and the ARPA), (II) addresses certain frequently asked questions that Proskauer attorneys have addressed, and (III) provides a brief overview of the Federal Reserve's Paycheck Protection Program Lending Facility, which is aimed at helping participating lenders originate more loans under the PPP loan for the many businesses, non-profits, and other eligible organizations in need of financial relief as a result of COVID-19.

This client alert will be updated to reflect any further changes in the key terms of the PPP resulting from any new legislation, rules, and guidance issued by the Federal government. While we have addressed below the principal criteria of the program and will endeavor to update this alert regularly, it is not possible to cover all of the rules and guidance published by the SBA and Treasury. THIS ALERT IS INTENDED TO BE A HELPFUL RESOURCE, BUT SHOULD NOT BE VIEWED AS LEGAL ADVICE FOR ANY SPECIFIC SITUATION. THIS ALERT IS UPDATED AS OF APRIL 8, 2021.

I. Key Terms of the Paycheck Protection Program

- **Maximum Loan Amount:** Under the CARES Act, a borrower's loan amount is equal to the lesser of: (i) 2.5x trailing 12 month average monthly payroll costs;[iv] and (ii) $10 million. Per the EAA Updated Rules (updating prior PPP Rules), borrowers are permitted to use any of (i) the 12 month period prior to the date on which the loan is made, (ii) calendar year 2019, or (iii) calendar year 2020 as the base period (though in our experience, most borrowers have

and will utilize calendar year 2019 or 2020). The SBA has updated its step-by-step "How to Calculate Loan Amounts" guide for calculating the maximum loan amounts based on the business type of an applicant in the EAA Updated Rules (III.B.4). In determining a borrower's average monthly payroll costs, a borrower should be consistent in the period it utilizes (i.e., a borrower should not mix 2019 and 2020 numbers). (*See below* for specific guidance on calculating total average monthly payroll costs for **Seasonal Employers** and **Self-Employed Applicants** and guidance as to **Partnerships**).

- **Single Corporate Group Cap:** The Interim Final Rule published on April 30, 2020 implemented a maximum cap of $20 million on the total amount of PPP loans that a "single corporate group" can receive. Businesses are part of a single corporate group if they are majority owned, directly or indirectly, by a common parent. This rule applies to all loans not fully disbursed by a lender – as opposed to those spent by a borrower – as of April 30, 2020 (and to the undisbursed portion of any partially disbursed loans). SBA affiliation rules are disregarded and "[b]usinesses are subject to this limitation even if the businesses are eligible for the waiver-of-affiliation provision under the CARES Act or are otherwise not considered to be affiliates under SBA's affiliation rules." Consequently, this cap applies to businesses that otherwise benefit from the affiliation waivers (including those in the accommodations and food services sector with North American Industry Classification System (NAICS) code beginning with 72).

  An applicant must (i) notify a lender if it has applied for or received PPP loans in excess of the $20 million cap and (ii) withdraw or request cancellation of any pending PPP loan application or approved PPP loan that would cause the applicant to exceed such cap. Failure to deliver such notice and to withdraw/request cancellation is deemed use of PPP funds for an unauthorized purpose and the PPP loan would be ineligible for forgiveness. While not expressly stated in the Interim Final Rule, additional penalties (criminal and civil) may apply to applicants who fail to comply with such requirements and retain or receive PPP loan proceeds in excess of the cap.

  The EAA Updated Rules suggest that Second Draw Loans received by PPP borrowers that are part of a single corporate group will not be counted toward that single corporate group's $20 million cap. However, all Second Draw Loans received by PPP borrowers that are part of a single corporate group will be subject to a $4 million cap as described below.

- **Interest Rate:** While the CARES Act provides (and the Economic Aid Act affirmed) a maximum permitted interest rate of 4.00%, the EAA Updated Rules reaffirm the actual interest rate on PPP loans is **1.00%** per annum**.** The Economic Aid Act clarifies that interest is to be calculated on a non-compounding, non-adjustable basis. Such interest rate and non-compounding, non-adjustable terms apply to all go-forward PPP loans and existing PPP loans if agreed upon by the PPP lender and eligible recipient.

- **Payment Deferral:** All principal, interest, and fees on the PPP loan may be deferred until the date on which the determined forgiveness amount is remitted to the lender so long as the borrower submits a forgiveness application within 10 months of the end of its forgiveness covered period. If a borrower fails to apply for forgiveness of a covered loan within 10 months after the last day of the covered period, such borrower must make payments of principal, interest, and fees beginning no earlier than 10 months after expiration of the covered period.[v] The PPPFA implemented the above deferral period (which period originally could expire after 6 months) and FAQ 52 confirms that such extension of the deferral period applies automatically to all PPP loans such that no modification of a promissory note is necessary.

- **Loan Maturity:** PPP loans mature after 5 years, if made on or after June 5, 2020 (the enactment of the PPPFA), or 2 years, if made prior to June 5, 2020. The borrower and the lender of pre-June 5, 2020 loans may amend the terms of existing PPP loans to reflect the longer maturity date. Under the PPP Rules, the date a PPP loan is made (at least, for purposes of determining the maturity date) is deemed to be the date on which the SBA assigned a loan number to that loan.

- **Collateral/Personal Guarantee:** No collateral or personal guarantee is required.

- **Eligibility**:

  - **Generally:** Eligible applicants (assuming they meet applicable size and other eligibility requirements listed below) include:

    1. business concerns;
    2. 501(c)(3) non-profit organizations;
    3. tax-exempt veterans organizations (501(c)(19));
    4. tribal business concerns (described in §31(b)(2)(C) of the Small Business Act);
    5. sole proprietors, independent contractors, and other self-employed individuals;
    6. a business assigned to the "accommodation and food services" sector (NAICS code beginning with 72);
    7. electric cooperatives exempt from federal income taxation under 501(c)(12) of the Internal Revenue Code as eligible as "a business entity organized for profit" (added under the PPP Rules on May 14, 2020);
    8. cooperative housing corporations (added by the Economic Aid Act);
    9. news organizations that are majority owned or controlled by a NAICS code 511110 (Newspaper Publishers) or 5151 (Radio or Television Broadcasting) business or non-profit public broadcasting entity with a trade or business under such NAICS code (expressly added by the Economic Aid Act);
    10. 501(c)(6) non-profit organizations (added by the Economic Aid Act);
    11. destination marketing organizations (added by the Economic Aid Act);
    12. certain debtors in a bankruptcy proceeding (although generally if an applicant is a debtor in a bankruptcy proceeding it would be ineligible, the Economic Aid Act moderates this restriction for certain business concerns);
    13. all other 501(c) organizations not listed above, except 501(c)(4) organizations (i.e. social welfare organizations) (added by the ARPA); and
    14. internet publishing organizations (added by the ARPA).

  An applicant must have been in operation on February 15, 2020 and either (A) had employees for whom salaries and payroll taxes were paid, or (B) paid independent contractors (as reported on Form 1099-MISC). A seasonal business will be considered to

have been in operation as of February 15, 2020, if the business was in operation for any 12-week period between May 1, 2019 and September 15, 2019.[vi]

An individual applicant is eligible if such individual (i) has self-employment income (such as an independent contractor or sole proprietor), (ii) was in operation on February 15, 2020, (iii) filed or will file a Form 1040 Schedule C for 2019, and (iv) has a principal place of residence in the United States.

Further, if a business was in operation on February 15, 2020, but has since changed ownership, it may apply for a PPP loan (assuming it is otherwise eligible). Similarly, if a change in ownership is effectuated through a sale of substantially all assets of a business that was in operation on February 15, 2020, the business acquiring the assets may apply for a PPP loan, even if the change in ownership results in a new TIN and even if the acquiring business was not in operation on February 15, 2020.

As noted above, the Economic Aid Act and the ARPA specifically identified as PPP loan-eligible the following, and established additional eligibility parameters specific to such entities/organizations:

- ***Housing Cooperatives*** – Cooperative housing corporations as defined in section 216(b) of the Internal Revenue Code of 1986 that employ not more than **300 employees**. In assessing eligibility, the SBA's affiliation rules apply to Housing Cooperatives.

- ***News Organizations*** – Business concerns, which expressly include any individual station licensed by the FCC under title III of the Communications Act of 1934, and any public broadcasting entity (as defined in section 397(11) of the Communications Act of 1934)[vii] if the following eligibility requirements are satisfied: (i) it is majority owned or controlled by a business concern that is assigned a NAICS code beginning with 511110 (Newspaper Publishers) or 5151 (Radio or Television Broadcasting) or, with respect to a non-profit public broadcasting entity, has a trade or business that falls under such a code; (ii) it employs not more than 500 employees (or the applicable NAICS size standard) per physical location[viii]; and (iii) it makes a good faith certification that loan proceeds will be used to support expenses in producing or distributing locally focused emergency information. Such borrowers are referred to herein as "News Entities." While such for profit News Entities were already covered as "business concerns" under one of the existing categories, the principal basis for the express addition is to recognize an individual "station" as an eligible applicant (i.e., an individual "concern" for purposes of the PPP) and to exempt such News Entities from the SBA's affiliation rules, thereby allowing such News Entities that may be part of a larger corporate group to receive PPP loans.

- ***501(c)(6) Organizations (Generally)*** – A 501(c)(6) organization is eligible to receive a PPP loan so long as: (i) it is not a professional sports league or organization that has a purpose of promoting or participating in a political campaign or other activity; (ii) such organization does not receive more than 15% of its receipts from lobbying activities; (iii) lobbying activities of the organization do not comprise more than 15% of the total activities of the organization; (iv) the total cost of the organization's lobbying activities did not exceed $1,000,000 during the most recent tax year of the organization that ended prior to February 15, 2020; and (v)

the organization does not employ more than **300 employees**. In assessing eligibility, the SBA's affiliation rules apply to an (otherwise eligible) 501(c)(6) organization. Such borrowers are referred to herein as "Eligible 501(c)(6) Organizations."

- ***Destination Marketing Organizations*** – **"**Destination Marketing Organizations," which are organizations that either are (a) described in 501(c) of the Internal Revenue Code and exempt from taxation under 501(a) of the Code, or (b) quasi-governmental entities or State political subdivisions, in each case, that are (i) engaged in marketing and promoting communities and facilities to business and leisure travelers through assisting the location of meeting and convention sites; providing travel information on area attractions, lodging accommodations, restaurants, and maps; and organizing group tours of local historical, recreational, and cultural attractions or (ii) engaged in and derive the majority of their operating budget from revenue attributable to providing live events, are eligible to receive PPP loans if: (w) the organization does not receive more than 15% of its receipts from lobbying activities; (x) lobbying activities of the organization do not comprise more than 15% of the total activities of the organization; (y) the total cost of the organization's lobbying activities did not exceed $1,000,000 during the most recent tax year of the organization that ended prior to February 15, 2020; and (z) the organization does not employ more than **300 employees**. In assessing eligibility, the SBA's affiliation rules apply to Destination Marketing Organizations.

- ***Businesses in Bankruptcy*** – The Economic Aid Act (Sec. 320) amends section 364 of the Bankruptcy Code, which governs post-petition financing, to permit a Bankruptcy Court (i) to authorize a debtor to obtain a PPP loan after notice and a hearing, and (ii) like the treatment afforded to a DIP lender, to grant "superpriority" administrative expense status to such claim if the SBA does not otherwise forgive the PPP loan. Importantly, because Sec. 320 only applies to those debtors authorized to operate in bankruptcy under section 1183, 1184, 1203, 1204, or 1304 of title 11, thereby excluding debtors authorized to operate under section 1106 and 1107 (i.e., chapter 11), it limits PPP loan eligibility to debtors in subchapter V (small businesses with no more than $7.5 million of debt), chapter 12 (family farmers or fishermen), and chapter 13 (individuals).

  While Bankruptcy Code section 1129(a)(9)(A) does not permit a debtor to confirm a plan of reorganization unless it pays all administrative claims in full in cash, the Economic Aid Act further provides that an eligible debtor can confirm a plan of reorganization that implicates an administrative expense claim derived from a PPP loan without paying the claim in full in cash, so long as the plan proposes to make payments on account of such claim when due under the terms of the PPP loan. Moreover, the Economic Aid Act also authorizes an eligible debtor to obtain a PPP loan on such terms even if a contract or loan agreement expressly prohibits the debtor from doing so. When read together, these amendments give enormous flexibility to certain, but not all, businesses seeking to restructure while also availing themselves of PPP benefits.

  In contrast, the SBA rules governing ineligible businesses have remained the same, providing that if a PPP applicant or "its owner" is a debtor in a bankruptcy proceeding at the time it submits an application or at any time before the loan is disbursed, the applicant is ineligible to receive the PPP loan, and if the applicant

or its owner becomes a debtor in a bankruptcy proceeding after submitting the application but before the PPP loan is disbursed, the applicant or its owner must cancel the application. Notably, these rules do not address what an applicant needs to do if it received the PPP loan before it or its owner became a debtor in bankruptcy. Question 59 of the FAQs provides that if an eligible debtor filed for bankruptcy protection after disbursement of the PPP loan, that debtor is eligible for loan forgiveness, provided it meets all other criteria for loan forgiveness, which supports the argument that a PPP loan can be obtained at any time before bankruptcy proceedings commence.

Consequently, there is a clear conflict between provisions of the Economic Aid Act and the EAA Updated Rules. The SBA will need to resolve that conflict so that there is clear guidance for businesses in bankruptcy for which the Economic Aid Act provided relief as to their ability to access PPP funds.

Question 60 of the FAQs provides that borrowers that received a First Draw PPP loan and filed for bankruptcy protection after disbursement of the First Draw PPP loan are not eligible to apply for a Second Draw PPP loan.

For guidance on when a borrower who has previously filed for bankruptcy protection is no longer considered to be "presently involved in any bankruptcy" proceeding, see **Question 12** in Section IV below.

- ***501(c) Organizations*** – Other than 501(c)(3), (4), (6) or (19) organizations, a 501(c) organization is eligible to receive a PPP loan so long as: (i) such organization does not receive more than 15% of its receipts from lobbying activities; (ii) lobbying activities of the organization do not comprise more than 15% of the total activities of the organization; (iii) the total cost of the organization's lobbying activities did not exceed $1,000,000 during the most recent tax year of the organization that ended prior to February 15, 2020; and (iv) the organization does not employ more than **300 employees**.

- ***Internet Publishing Organizations*** – Business concerns or other organizations that were not eligible to receive a PPP loan before the date of enactment of the ARPA, are assigned a NAICS code of 519130, certify in good faith as Internet-only news publishers or Internet-only periodical publishers, and are engaged in the collection and distribution of local or regional and national news and information ("Internet Publishing Organizations") are eligible to receive a PPP loan if the following eligibility requirements are satisfied:

    - The business concern or organization employs not more than 500 employees (or the size standard established by the Administrator for that North American Industry Classification code) per physical location; and

    - The business concern or organization makes a good faith certification that proceeds of the loan will be used to support expenses at the component of the business concern or organization that supports local or regional news.

- **Ineligible Industries:** An applicant is not eligible if its business is in an ineligible industry or otherwise described as ineligible under 13 C.F.R. § 120.110 (also detailed in the SBA's Standard Operating Procedure (SOP) 50-10-6), except where there is an express

exception under the CARES Act (such as for certain non-profits) or the PPP Rules. Key ineligible industries include businesses primarily engaged in lending or investment and passive investment in real estate.

The PPP Rules contain notable changes and clarifications to the scope of ineligible industries or businesses that would otherwise be ineligible under traditional SBA rules:

- ***Hedge Funds and Private Equity Firms are Not Eligible*** – Hedge funds and private equity firms are ineligible to receive PPP loans as they are "engaged in investment or speculation." Portfolio companies of private equity funds *may* still be eligible if they meet applicable size standards after application of the affiliation rules and can make (after careful consideration) the "necessity" certification (each discussed below).

- ***Legal Gambling Businesses are Eligible*** – Businesses that derive revenue from legal gambling activities are now eligible for PPP loans regardless of the amount of the business's revenue that is derived from gambling activities (as 13 C.F.R. § 120.110(g) no longer applies to the PPP).

- ***Certain Government-Owned Hospitals are Eligible*** – A state or local government-owned hospital that would otherwise be ineligible (under 13 C.F.R. § 120.110(j)) as a government-owned entity, is now eligible for a PPP loan if the hospital receives less than 50% of its funding from state or local government sources, exclusive of Medicaid.

- ***Most Businesses in Bankruptcy are Not Eligible*** – *See* ***Businesses in Bankruptcy*** above.

- ***PPP Lender Affiliate Restrictions*** – An Interim Final Rule published on April 20, 2020 narrowed the limitations in 13 C.F.R. 120.110 and 120.140 to provide that an (otherwise eligible) business owned (in whole or in part) by an outside director or equityholder of less than 30% of the equity in a PPP lender is permitted to receive a PPP loan from such PPP lender (so long as such business follows the same process as any similarly situated customer). Officers and key employees of a PPP lender (or businesses owned thereby) may not receive a PPP loan from the lender with which they are employed.

Other ineligible entities specifically identified in the PPP Rules include household employers (*i.e.*, individuals who employ household employees such as nannies or housekeepers); businesses 20% or more of which are owned by persons who are incarcerated, under indictment, or subject to other means by which formal criminal charges are brought, or have been convicted of a felony in the last five years involving fraud, bribery, embezzlement, or a false statement on a loan application or application for federal financial assistance; and businesses owned or controlled by any person that has ever obtained an SBA or other Federal loan (other than Federal student loans in general) that is currently delinquent or has defaulted within the last seven years and caused a loss to the government.

The Economic Aid Act and the EAA Updated Rules provides that businesses and organizations in the following additional categories are now also expressly ineligible to receive a PPP loan:

- ***Not in Operation on February 15, 2020*** – A business/organization that was not in operation on February 15, 2020 (to be clear, this change reaffirms the existing limitation and has retroactive effect to the passage of the CARES Act as set forth in the Economic Aid Act).

- ***Shuttered Venue Operators Grant Recipient*** – A person or entity that receives a "Shuttered Venue Operators" grant under section 24 of the Economic Aid Act. Note that under the ARPA, if a person or entity applied for and received a PPP loan on or after December 27, 2020, any such PPP loan will reduce the Shuttered Venue Operators grant by the amount of the PPP loan. (For more information on the Shuttered Venue Operators program, please see our publication "Front and Center: New SBA Grant Program for Shuttered Venue Operators".)

- ***Public Issuer*** – Beginning on December 27, 2020, an issuer of publicly traded securities registered on a national exchange (this would not apply to such issuers who have already received a PPP loan under the CARES Act). The Economic Aid Act clarifies that the fact that a News Entity's affiliate (including any entity that owns or controls a News Entity) is a publicly-traded news organization does not render the News Entity itself ineligible. While the text of the Economic Aid Act suggests that this carve-out is intended to apply to publicly-traded news organizations, it is ultimately unclear and will need to be clarified in SBA rule making or FAQs.

- ***Professional Sports League or Political Organization*** – A 501(c)(6) organization that is a professional sports league or organization that has a purpose of promoting or participating in a political campaign or other activity.

- ***Federal Official Ownership*** – A business concern of which the President, the Vice President, the head of an Executive Department, or a Member of Congress (or the spouse of such person as determined under applicable common law), directly or indirectly, holds a controlling interest.

  Under the EAA Updated Rules, borrowers with PPP loans made *before* December 27, 2020 must disclose any such controlling interest by not later than January 26, 2021 if any application was submitted for forgiveness *prior to* December 27, 2020 and within 30 days for all other forgiveness applications. While the language of the EAA Updated Rules is not clear, this disclosure requirement also appears to apply if any such government official is or holds a controlling interest in the principal executive officer or any individual performing a similar function of the borrower.

- ***Business that has been Permanently Closed*** – An entity that has gone out of business and has no intention of reopening is barred from receiving a PPP loan. The EAA Updated Rules make clear that an otherwise qualified borrower that has temporarily closed or suspended its business, but intends to reopen, remains eligible for a PPP loan.

- **Size Standard:** An applicant (taking into account its affiliates) must either:

  - ***Existing Size Standards*** – qualify as a "small business concern" by meeting the SBA's existing SBA size standards for the applicable NAICS code, which are based on either employee headcount (full-time, part-time, or other basis) or 3-year average annual gross receipts;

- ***Alternative Size Standard*** – qualify as a "small business concern" by meeting the SBA's "alternative size standard," which requires that the applicant (together with its affiliates) have not more than **$15 million** in tangible net worth *and* not more than **$5 million** in average net income after Federal income taxes (excluding any carry-over losses) for the 2 full fiscal years before the date of the application (13 C.F.R. § 121.301(b)(2) is instructive as to how to calculate net income after Federal income taxes for pass-through entities);

- ***Employee Headcount Standard*** –

    - generally have (together with its affiliates) not more than 500 employees (on a full-time, part-time, or other basis),

    - in the case of Eligible 501(c)(6) Organizations, Destination Marketing Organizations, and Housing Cooperatives, not more than 300 employees (on a full-time, part-time, or other basis),

    - in the case of Eligible 501(c)(3) Organizations, not more than 500 employees per physical location of the organization,

    - in the case of other Eligible 501(c) Organizations (other than (501(c)(3), (4), (6) or (19) organizations), not more than 300 employees per physical location of the organization,

    - in the case of Eligible Internet Publishing Organizations, not more than 500 employees, or the size standard established by the Administrator for that North American Industry Classification code, per physical location, **if** the organization is majority owned or controlled by a business concern or organization that is assigned a North American Industry Classification System code of 519130, or

    - in the case of Eligible Electric and Telephone Cooperatives, not more than 300 employees per physical location (and these entities are not permitted to use the existing SBA size standards for their industry or the SBA's alternative size standard).

- ***Accommodations and Food Services*** – be a business assigned to the "accommodation and food services" sector (NAICS code beginning with 72) having not more than 500 employees per physical location.

- **Affiliation:** When determining whether any of the above size standards are met, the SBA's existing affiliation rules require a business to aggregate the number of its employees, receipts, or other applicable metric with that of its foreign and domestic affiliates. Applicants and entities are affiliates when one controls or has the power to control the other or such entities are under common control. Control is broadly defined in the SBA's regulations, and encompasses affirmative and negative control rights, as well as equity-based and contractual control rights (including affiliation based on a management agreement). The SBA has confirmed that the pre-2020 version of 13 C.F.R. § 121.301(f), the affiliation rule for 7(a) loans, applies to the PPP. Relatedly, the SBA and Treasury have issued Affiliation Guidance with respect to the affiliation rules that apply to the PPP. There are some exceptions to the

application of the SBA's existing affiliation rules that are specific to the Paycheck Protection Program:

- ***CARES Act Exceptions*** – Under the CARES Act, the SBA's affiliation rules are waived for businesses in the accommodation and food service sector with an NAICS code beginning with 72, franchises assigned a franchise identifier code by the SBA, and businesses that receive assistance from an approved small business investment company under § 301 of the Small Business Investment Act of 1958 (*e.g.*, SBIC portfolio companies).[ix] As a result of this exception, each hotel or restaurant location owned by a parent business (held within a separate legal entity) that employs not more than 500 employees can apply for a separate PPP loan, provided it uses a unique EIN. However, this waiver applies only when determining eligibility for an applicant business with the 72-code. The affiliation exemption does not apply when determining eligibility of an applicant that is not in such sector. Such applicant would be required to take into account the employees, receipts, or other applicable metric of all of its affiliates, *including* those operating in the accommodations or food service sector.

- ***PPP Rules Exceptions*** –

  - ***Faith-Based Organizations*** – Under the PPP Rules, affiliation rules are waived for faith-based organizations where the application of such rules would "substantially burden [such an] organization's religious exercise."

  - ***Employee Stock Option Ownership*** – Under the PPP Rules, a business that participates in an Employee Stock Ownership Plan (ESOP) does not trigger affiliation between the business and the ESOP.

- ***Statutory Exceptions*** – Under the SBA's existing regulations, the exceptions to the affiliation rules in 13 C.F.R. § 121.103(b) (but not the exception in 121.103(b)(5), which does not apply to 7(a) loans) continue to apply in the context of the PPP. While these exceptions should be reviewed in connection with any affiliation analysis, they are narrow and will not benefit most businesses (unless owned or controlled by certain tribal organizations or small business investment companies).

- ***Economic Aid Act and the ARPA Exceptions (News Entities and Internet Publishing Organizations)*** – Under the Economic Aid Act and the ARPA, the affiliation rules are waived for eligible News Entities and Internet Publishing Organizations. As a result, a News Entity location (*i.e.*, individual radio station) or an Internet Publishing Organization location that employs not more than 500 employees can apply for a separate PPP loan.

- **Calculating Employee Headcount:** Borrowers should use either of the following methods for purposes of determining employee headcount: (i) average employment over the same time periods as used for payroll costs (12 month-period preceding the loan date, calendar year 2019, calendar year 2020, or applicable period for seasonal businesses) to determine number of employees, for the purposes of applying an employee-based size standard; or (ii) average number of employees per pay period in the 12 *completed* calendar months prior to the date of the loan application (or the average number of employees for each of the pay periods that the business has been operational, if less than 12 months).

  - ***Inclusion of Foreign Employees*** – In accordance with 13 C.F.R. § 121.301(f)(6), for both the PPP's 500 or fewer employee size standard and businesses otherwise seeking

to qualify as a "small business concern" on the basis of the employee-based size standard, an applicant **must count all of its employees and the employees of its U.S. and foreign affiliates**, absent a waiver of or an exception to the affiliation rules.[x]

- **Necessity**: Applicants are required to certify that the "*current economic uncertainty makes this loan request necessary to support the ongoing operations of the Applicant*." All applicants (but especially **larger companies and portfolio companies of private equity sponsors**) should carefully review and be thoughtful about the implications of making this certification, including how it speaks to the applicant's economic viability and the message it communicates to investors and the market). **When making a "necessity" assessment, applicants should create a thoughtful and detailed record supporting their determination and the process employed in that assessment.**

  - **Other Sources of Liquidity:** The SBA has clarified (in Questions 31 and 37 of the SBA FAQs) that while the CARES Act waives the "credit elsewhere" requirement, borrowers must nonetheless carefully review and make the "necessity" certification in good faith. In so doing, borrowers must take "into account their current business activity and their ability to access other sources of liquidity sufficient to support their ongoing operations in a manner that is not significantly detrimental to the business." This applies to both publicly traded and private companies.

  - **Large/Public Companies:** As a response to the much-reported receipt of PPP loans by certain publicly traded companies, the SBA previously clarified that it is **unlikely** that a company with substantial market value and access to capital markets will be able to make the required certification in good faith, and such a company should be prepared to demonstrate to the SBA, upon request, the basis for its certification. While such guidance remains relevant to public companies that may have previously received a PPP loan, as noted above, under the Economic Aid Act, public companies will not be eligible for PPP loans going forward.

  - **Retraction and Safe Harbor:** Any borrower (whether publicly-traded or privately-owned) that applied for a PPP loan *prior* to April 24, 2020 and repaid the loan in full by **May 18, 2020** is deemed to have made the required certification in good faith.

  - **Review of the Necessity Certification:**

    - ***Borrowers of Less than $2 million*** – As announced in Question 46 of the FAQ (published on May 13, 2020), a borrower that, together with its affiliates, received PPP loans with an original principal amount of **less than $2 million** will be deemed to have made the "necessity" certification in good faith.

    - ***Borrowers of $2 million or Greater*** – Question 39 of the FAQ (published on April 29, 2020) provides that the SBA will review all loans in excess of $2 million, *in addition to other loans as appropriate*, following the lender's submission of the borrower's loan forgiveness application. Question 46 of the FAQ clarifies that a borrower's "necessity" certification will be assessed as part of such review, and if the SBA determines in the course of its review that the borrower lacked an adequate basis for the "necessity" certification, the SBA will (i) seek repayment of the outstanding PPP loan balance and (ii) inform the lender that the borrower is not eligible for loan forgiveness. So long as the borrower repays the loan following

such notification, the SBA will not pursue administrative enforcement or referrals to other agencies based on its determination with respect to the "necessity" certification (though the SBA may of course refer any other issues identified).[xi]

- ***Paycheck Protection Program Loan Necessity Questionnaire*** – In addition, in late 2020, the SBA produced loan necessity questionnaires for each of for-profit borrowers (SBA Form 3509) and non-profit borrowers (SBA Form 3510) that must be completed and submitted by each PPP borrower that together with its affiliates received PPP loans with an original principal amount of $2 million or greater to such PPP borrower's lender within ten business days of receipt of such from such lender. Failure to complete such forms and provide required supporting documents may result in SBA's determination that a PPP borrower was ineligible for its PPP loan, its PPP loan amount, or any forgiveness amount claimed. Each of these necessity questionnaires includes a portion titled "liquidity assessment" which includes specific questions about available funds immediately prior to a borrower's PPP loan application, and the use of funds between March 13, 2020 and the end of a borrower's loan forgiveness period (including as to any dividends or distributions, pre-payment of debt, compensation to employees in excess of $250,000, and how many were so compensated).

  While FAQ 53 states that the information that a borrower provides on the questionnaires will help the SBA assess the borrower's certification in its loan application that "[c]urrent economic uncertainty makes this loan request necessary to support the ongoing operations of the Applicant," as required by the CARES Act, the questionnaires do not specify whether any of the borrower's responses will be dispositive as to whether a PPP loan was in fact necessary for a borrower completing the form, and consequently, there remains uncertainty around whether a borrower can make the certification of need when it may have access to other sources of liquidity. What constitutes "liquidity" or when would the use of such liquidity be "significantly detrimental"? What about a case where a borrower's business has no cash or other readily available sources of liquidity, but the borrower's owners, such as private equity or other funds, may have or be able to access such liquidity? This ambiguity is particularly problematic for hotels, restaurants, and other 72-code businesses that have faced severe reduction or even elimination of all revenues and that are owned by private equity sponsors, but are exempted from the affiliation rules, and are thus eligible to receive PPP loans if they are able to make the "necessity" certification. Both the legal and the public relations "judgments" will be made in hindsight, which leaves borrowers and their sponsors facing difficult choices in a crisis without any clear end.

- **Eligible Uses:** PPP loan proceeds may be used for:

  1. payroll costs, which include, among others, (i) costs related to the continuation of group health care, life, disability, vision, or dental benefits during periods of paid sick, medical, or family leave, and group health care, life, disability, vision, or dental insurance premiums, and (ii) employee salaries, commissions, or similar compensations (payroll costs is discussed in further detail below);

2. payments of interest on any mortgage (but not prepayment of or payment of principal);
3. rent (including under a lease agreement);
4. utility payments;
5. interest on any other debt obligations incurred before February 15, 2020;
6. refinancing an SBA EIDL Loan made between January 31, 2020 and April 3, 2020;
7. covered operations expenditures (added by the Economic Aid Act);
8. covered property damage costs (added by the Economic Aid Act);
9. covered supplier costs (added by the Economic Aid Act); and
10. covered worker protections expenditures (added by the Economic Aid Act).

The CARES Act provides that loan proceeds can also be used for any allowable use for which a 7(a) loan can be applied under the Small Business Act, which uses are set forth in 13 C.F.R. § 120.120 and include, *e.g.*, inventory, supplies, and working capital. However, the PPP Rules list as permitted only those uses detailed above, and it remains unclear whether the SBA is restricting permitted uses to *only* those that are expressly listed above. Note further that some of the items listed above are not forgiveness-eligible, and any additional allowable uses not specifically listed in the CARES Act or the PPP Rules are not forgiveness-eligible.

In addition to the expansion of Payroll Costs (which are permitted PPP loan uses), the Economic Aid Act expands upon the categories of permitted uses for PPP loans to include the following (each of which is also a forgiveness-eligible use):

- ***Covered Operations Expenditures*** – Payments made for any business software or cloud computing service that facilitates business operations, product or service delivery, the processing, payment, or tracking of payroll expenses, human resources, sales and billing functions, or accounting or tracking of supplies, inventory, records, and expenses.

- ***Covered Property Damage Costs*** – Costs related to property damage and vandalism or looting due to public disturbances that occurred during 2020 not covered by insurance or other compensation.

- ***Covered Supplier Costs*** – Expenditures made by an entity to a supplier of goods for the supply of goods that (1) are essential to the operations of the entity at the time at which the expenditure is made; and (2) are made pursuant to a contract, order, or purchase order (i) in effect at any time before the covered period with respect to the applicable covered loan; or (ii) with respect to perishable goods, in effect before or at any time during the covered period with respect to the applicable covered loan.

- ***Covered Worker Protection Expenditures*** – Operating or capital expenditures to facilitate the adaptation of the business activities of an entity to comply with requirements established or guidance issued by the Department of Health and Human Services, the Centers for Disease Control, or the Occupational Safety and Health Administration, or any equivalent requirements established or guidance

issued by a State or local government beginning on March 1, 2020 and ending on the date on which the COVID-19 national emergency expires (per executive action by the President). Such expenditures may include purchase, maintenance, or renovation of assets that create or expand:

- a drive-through window facility;
- indoor, outdoor, or combined air or air pressure ventilation or filtration system;
- a physical barrier such as a sneeze guard;
- an expansion of additional indoor, outdoor, or combined business space;
- an onsite or offsite health screening capability;
- other assets relating to expenditures made to facilitate the adaptation of a business or entity in compliance with Covered Worker Protection Expenditures requirements as determined by the SBA in consultation with the Secretary of Health and Human Services and the Secretary of Labor; or
- the purchase of materials as described in section 328.103(a) of title 44, Code of Federal Regulations, or any successor regulation, filtering face piece respirators approved by the National Institute for Occupational Safety and Health, including those approved only for emergency use authorization, and/or other kinds of personal protective equipment, as determined by the SBA Administrator in consultation with the Secretary of Health and Human Services and the Secretary of Labor.

Note, however, that Covered Work Protection Expenditures do not include residential property or intangible property. In addition, the Economic Aid Act expressly prohibits use of PPP loan proceeds for lobbying activities.[xii]

- **60% Payroll Cost Threshold:** The PPP Rules, as amended by the Interim Final Rules published June 11, 12, 17, 22, and 24 (the "PPPFA Revision Rules"), require borrowers to use at least 60% of PPP loan proceeds for payroll costs (the CARES Act itself does not impose such a requirement). Previously, the PPP Rules required 75% of the PPP loan proceeds be used for payroll costs. The Economic Aid Act does not change this requirement.

- **Independent Contractors:** A business **cannot** include independent contractors as "employees" either for purposes of calculating the loan amount (i.e., with payroll cost calculations) or amount of loan forgiveness. Independent contractors can themselves apply for PPP loans.

- **Required EIDL Refinancing:** On June 22, 2020, the SBA published additional guidance for when PPP loan proceeds must be used to refinance an EIDL loan. This guidance describes three scenarios:

  - An EIDL loan may not be refinanced with a PPP loan when the PPP borrower received the EIDL loan before January 31, 2020 or after April 3, 2020.

- An EIDL loan is not required to be refinanced with a PPP loan when (a) the PPP borrower received funds from an EIDL loan from January 31, 2020 through April 3, 2020, and (b) the PPP borrower used the EIDL loan for purposes other than payroll costs.

- A PPP loan must be used to refinance the full amount of the EIDL loan when (a) the PPP borrower received funds from the EIDL loan from January 31, 2020 through April 3, 2020, and (b) the PPP borrower used the EIDL loan funds to pay payroll costs.

The Economic Aid Act also provides for a renewal and expansion of the EIDL loan program in response to the COVID-19 pandemic, and it is reasonable to expect that SBA guidance on this topic may be updated in the future. For information regarding the renewed and expanded EIDL program under the Economic Aid Act, *see* Where Are We Now? – Paycheck Protection Program Redux.

- **Payroll Costs for Self-Employed Applicants:** Self-employed borrowers who filed (or are eligible to file) a Form 1040 Schedule C for 2019 or 2020 may use loan proceeds for: (i) (A) for borrowers that use *net profit* to calculate loan amount, owner compensation equal to total average monthly net profit for 2019 or 2020 (whichever year was used to calculate the maximum loan amount) up to a maximum annualized amount of $100,000, or (B) for borrowers that use *gross income* to calculate loan amount, proprietor expenses (business expenses plus owner compensation) equal to total average monthly gross income for 2019 or 2020 (whichever year was used to calculate the maximum loan amount) up to a maximum annualized amount of $100,000, provided that in no event shall the amount in (A) or (B) exceed $20,833; (ii) payroll costs to employees with a principal place of residence in the US (if any); (iii) mortgage interest, rent, or utility payments that can be claimed as a business expense deduction on Form 1040 Schedule C for 2019 or 2020; (iv) interest payments on any loan incurred prior to February 15, 2020; (v) refinancing of any EIDL obtained between January 31, 2020 and April 3, 2020; and (vi) covered operations expenditures, property damage costs, supplier costs, and worker protection expenditures. Further, the PPP Rules indicate that an applicant that did not claim (or was not entitled to claim) such mortgage interest, rent, or utility payments on its 2019 or 2020 Form 1040 Schedule C (whichever period is used) **cannot** use the loan proceeds for such expenses during the initial 24-week period (or 8-week period) following the first disbursement of the loan. The 40% limitation on non-payroll cost uses applies to self-employed applicants.

  Under the PPPFA Revision Rules and SBA Form 3508 the maximum amount of total payroll costs of a self-employed borrower that is forgivable with respect to a 24-week covered period is $20,833 and for an 8-week covered period is $15,385, even if the self-employed borrowers can pay themselves more from the PPP loan and remain under the $100,000 annualized cap. No changes to these amounts were expressly included in the EAA Updated Rules.

- **Student Workers:** Student workers generally count as employees unless (a) the applicant is an institution of higher education (as defined in Department of Education Federal Work-Study regulations) and (b) the student worker's services are performed as part of a Federal Work-Study Program or a substantially similar State or political subdivision program. Institutions of higher education must exclude all work study students when determining PPP loan eligibility and exclude payroll costs for work

study students from the calculation of payroll costs used to determine their PPP loan amount.

- **Payroll Costs**:

  - **Included**: "Payroll Costs" generally include the following compensation for employees (and not any independent contractors) whose principal place of residence is in the US: (i) salary, wage, commission, or similar compensation; (ii) cash tips or equivalents; (iii) payment for vacation, parental, family, medical, or sick leave; (iv) allowance for dismissal or separation; (v) payment required for the provision of group health care benefits, including group life, disability, vision, or dental insurance premiums (but excluding expenses for group health care benefits paid by employees (or beneficiaries of the plan) either pre-tax or after tax, such as the employee share of their health care premium); (vi) payment of any retirement benefit; and (vii) payment of state or local taxes assessed on employee compensation. The SBA has indicated that payroll costs are calculated on a gross basis without regard to federal taxes imposed or withheld.

    Additionally, the Economic Aid Act (as previously affirmed in an August 11, 2020 FAQ) expressly included group life, disability, vision, or dental insurance benefits as Payroll Costs and such addition is retroactive to the enactment of the CARES Act.

    - ***Calculating Total Average Monthly Payroll Costs to Determine a PPP loan Amount*** – Potential new PPP applicants are encouraged to refer to the step-by-step maximum loan amount calculation guide contained in the EAA Updated Rules.

      - ***Applicants Generally*** – Other than for the specific categories identified below, an applicant's payroll costs for purposes of calculating a PPP loan amount is based upon (i) the 12 months prior to the date of a PPP loan, (ii) 2019, or (iii) 2020, aggregate payroll costs for employees whose principal place of residence is the US.

      - ***Seasonal Employer's Loan Amount*** – The Economic Aid Act provides that seasonal employers can calculate average total monthly payroll costs using any 12-week period between February 15, 2019 and February 15, 2020. Further, the Economic Aid Act redefines a seasonal employer to mean an entity that does not operate for more than seven (7) months in any calendar year, or during the preceding calendar year had gross receipts for any six (6) months of that year that were not more than 33.33% of the gross receipts of such employer for the other six (6) months of that year.

      - ***Self-Employed Applicant Loan Amount*** – When calculating payroll costs for purposes of determining a borrower's loan amount, such compensation for self-employed applicants that filed (or will file) a Form 1040 Schedule C (or F) for 2019 or 2020 will be:

        (A) for self-employed applicants with no employees, equal to 2.5 times the average monthly net profit **or** gross income amount computed therein (subject to an annualized $100,000 cap), not to exceed $20,833; or

(B) for self-employed applicants with employees, equal to 2.5 times the average monthly net profit **or** gross income (less the self-employed applicant's employee payroll costs) amount computed therein (subject to an annualized $100,000 cap), not to exceed $20,833.

- For self-employed applicants that have employees, payroll costs for such employees are calculated using:
  - 2019 or 2020 gross wages and tips paid to such employees with a principal place of residence in the US (using 2019 or 2020 IRS Form 941 Taxable Medicare wages & tips from each quarter) *plus* any pre-tax employee contributions for health insurance or other fringe benefits excluded from Taxable Medicare wages & tips (net of any amounts paid to any individual employee in excess of $100,000 annualized cap); and
  - 2019 or 2020 employer group health, life, disability, vision, and dental insurance contributions and retirement contributions listed on the 2019 or 2020 Form 1040 Schedule C or F and state and local taxes assessed on employee compensation.

- ***Partnerships with General Operating Partners*** – Partners in a partnership may <u>not</u> submit a separate PPP loan application as a self-employed individual. Self-employment income of general active partners may be reported as a payroll cost on a PPP loan application filed by or on behalf of the partnership. The SBA's step-by-step maximum loan amount calculation guide confirms that payroll costs for self-employment income for individual U.S.-based general partners is calculated using 2019 or 2020 Schedule K-1 (IRS Form 1065) net earnings from self-employment (reduced by any section 179 expense deduction claimed, unreimbursed partnership expenses claimed, and depletion claimed on oil and gas properties) multiplied by 0.9235[xiii], subject to a $100,000 annualized cap (as prorated for the period during which the payments are made or the obligation to make the payments is incurred).

- ***Farmers and Ranchers*** – While not detailed here, the SBA has provided (and updated in the EAA Updated Rules) step-by-step guidance for calculating total monthly average payroll costs (and therefore the maximum PPP loan amount) for farmers and ranchers who operate as a sole proprietorship, independent contractor, or self-employed individual.

- **<u>Excluded</u>:** Payroll costs **<u>do not</u>** include: (i) cash compensation (*i.e.*, gross amount before deductions for taxes, employee benefits payments, and similar payments) of any individual employee in excess of an annual salary of $100,000, as prorated for the period during which the payments are made or the obligation to make the payments is incurred; (ii) federal income taxes imposed or withheld under chapters 21, 22, or 24 of the Internal Revenue Code of 1986 during the covered period (includes Federal Insurance

Contributions Act and Railroad Retirement Act taxes and income taxes required to be withheld from employees); (iii) qualified sick and family leave wages for which a credit is allowed under sections 7001 and 7003 of the Families First Coronavirus Response Act; and (iv) premiums taken into account in determining the credit allowed under section 6432 of the Internal Revenue Code of 1986[xiv]. In addition, forgiveness cannot be requested for any "qualified wages" that allowed the employer to claim ERTCs from "payroll costs" (see below under Tax Matters).

- **Period for Calculating Payroll Costs:** SBA guidance indicates that borrowers (other than self-employed applicants) can calculate their aggregate payroll costs using data either from the trailing 12 months, calendar year 2019, or calendar year 2020. Seasonal businesses may use average monthly payroll for the period between February 16, 2019 and June 30, 2019 *or* March 1, 2019 and June 30, 2019.

- **Ability for PPP Borrowers to Request an Increase in Loan Amount**:

  - ***Returned a Portion of PPP loan*** – if a borrower returned a portion of its PPP loan amount before December 27, 2020, such borrower may reapply for a PPP loan (and distinct from a Second Draw Loan) in an amount equal to the difference between the amount retained and such applicant's maximum loan amount; or

  - ***Declined a Portion of PPP loan*** – if a borrower declined to accept part of its PPP loan before December 27, 2020, such borrower may request that the PPP lender modify such borrower's PPP loan to increase the loan amount to the maximum amount for which such borrower is eligible.

  These provisions would apply even if the full loan had already been disbursed and a lender had already issued a Form 1502 with the SBA**.**

- **Loan Forgiveness:** On May 22, 2020 the SBA published Interim Final Rules on loan forgiveness (the "Forgiveness Rules"), which have been subsequently amended on June 22, 2020 and October 8, 2020. In addition, on May 22, 2020, the SBA published an Interim Final Rule on SBA loan review procedures and related borrower and lender responsibilities, on July 23, 2020, the SBA published Guidance on Procedures for Lender Submission of PPP loan Forgiveness Decisions to SBA and SBA Forgiveness Loan Reviews ("Review Rules"), and on August 4, 2020, the SBA published an FAQ on PPP loan Forgiveness. On January 19, 2021, the SBA and Treasury published an Interim Final Rule on loan forgiveness requirements and loan review procedures as amended by the Economic Aid Act, consolidating prior rules related to forgiveness and review of PPP loans including with respect to forgiveness of Second Draw Loans. On January 19, 2021, the SBA and Treasury released an updated version of SBA Form 3508, which implements the changes contained in the Economic Aid Act, a simplified forgiveness application, SBA Form 3508EZ, and a further simplified one page forgiveness application, SBA Form 3508S, specifically for borrowers with a PPP loan of $150,000 or less. SBA Forms 3508, 3508EZ and 3508S are referred to below together as the "forgiveness applications". With respect to eligibility to use SBA Form 3508EZ see **Question 9** below.

  - **Forgiveness Amount:** Under the PPP Rules, up to the entire principal amount and any accrued interest on a PPP loan is eligible for forgiveness if applied toward forgiveness-eligible uses. Generally, a borrower is **eligible** for a forgiveness amount that is the lesser of (i) its full PPP loan amount (no mention of interest), (ii) the sum of all forgiveness-eligible costs spent during the covered period as reduced for employee compensation and full-

time employee equivalent ("FTE") headcount reductions (discussed below), and (iii) the quotient obtained by dividing the amount of the loan used for payroll costs during the covered period and 0.6 (such that the amount forgiven is not less than 60% of such payroll costs).

- **Covered Period:** As revised under the Economic Aid Act, the covered forgiveness-eligible period begins on the date of the origination of the covered loan and ends on a date selected by the eligible recipient that occurs during the period beginning 8 weeks after origination and ending 24 weeks after the origination date (*i.e.*, a borrower can select a covered period between 8 and 24 weeks). As affirmed under the EAA Updated Rules, the Economic Aid Act does not alter the existing PPP Rules, which dictate whether certain expenses incurred or paid during a borrower's covered period are forgiveness-eligible.

    - ***Alternative Covered (Payroll) Period*** – The EAA Updated Rules remove the construct of an "alternative covered period" (previously included in the PPP Rules and forgiveness applications). The EAA Updated Rules do not expressly indicate whether such change is retroactive, such that if a borrower has *already* applied for (but not yet received) forgiveness utilizing the alternative covered period, it is unclear if such borrower would be required to update its forgiveness application. The expectation would be that the removal of the alternative covered period only applies to PPP loans made in 2021.

- **Forgiveness-Eligible Costs:** Forgiveness-eligible costs include payroll costs, interest payments on mortgages on real or personal property (*e.g.*, auto loans) existing before February 15, 2020 (but excluding interest on unsecured credit), rent under leases in place before February 15, 2020, and payments for utilities (including gas, water, telephone, internet access, transportation (including transportation utility fees assessed by state and local governments), and electricity (including supply charges, distribution charges, and other charges such as gross receipts taxes)), for which service began before February 15, 2020, in each case **incurred or paid** during a 24-week (or 8-week) covered period. Payments of rent or interest on leases and mortgages that existed prior to February 15, 2020 and were renewed or refinanced after such date are eligible for loan forgiveness as well. In addition, the Economic Aid Act expands upon eligible covered expenses to include (i) covered operations expenditures, (ii) covered property damage costs, (iii) covered supplier costs, and (iv) covered worker protection expenditures (described above).

  To receive loan forgiveness, a borrower must use at least **60%** of the loan amount for payroll costs. The PPP Rules (as reaffirmed in the EAA Updated Rules) interpret this requirement as a proportional limitation on the loan forgiveness amount such that 60% of the loan forgiveness amount requested must have been used on payroll costs, rather than a threshold requirement that 60% of the total loan amount must be used on payroll costs *before* a loan can be forgiven. If payroll costs represent less than 60% of the total loan forgiveness amount requested by a borrower, then the forgiveness amount is proportionately reduced until payroll costs constitute 60% of the total forgiveness amount. The Economic Aid Act does not alter the 60% payroll cost use requirement.

    - **Eligible Payroll Costs** – Payroll costs are considered paid on the day that paychecks are distributed or the borrower *originates* an ACH credit transaction. Payroll costs are considered incurred on the day that the employee's pay is earned. Payroll costs incurred but not paid during the borrower's last pay period of the covered period are eligible for forgiveness if paid on or before the next regular

payroll date. Otherwise, payroll costs must be paid during the covered period. Payroll costs for employees not performing work but still on the borrower's payroll are incurred based on the schedule established by the borrower (typically, each day that the employee would have performed work).

- **Costs of Furloughed Employees** – If a borrower pays furloughed employees their salary, wages, or commissions during the covered period, those payments are eligible for forgiveness as long as they do not exceed an annual salary of $100,000, as prorated for the period during which the compensation/payment is made or the obligation to pay is incurred (however, the reduced hours for such furloughed employees will impact the forgiveness amount under the current rules and Form 3508),

- **Increased Compensation/Bonuses** – An employee's hazard pay, commissions, tips and bonuses are eligible for loan forgiveness (as a supplement to salary or wages) so long as the employee's total compensation does not exceed $100,000 on an annualized basis.

- **Certain Owner-Employees/Partners/Self-Employed Individuals** – Forgiveness-eligible payroll costs for owner-employees (with an ownership stake of 5% or more in either a C- or S-corporation), partners, and self-employed individuals cannot exceed 2.5 months' worth of compensation received in the year used to calculate the PPP loan amount (that is, 2019 or 2020), capped at $20,833 per individual in total across all businesses. (Note that owner-employees with less than a 5% ownership stake in a C- or S-corporation are not subject to the owner-employee compensation rule contemplated in this section.) The individual's total compensation may not exceed $100,000 on an annualized basis, prorated for the applicable covered period (for example, if a borrower elects to use an eight-week covered period, the amount of loan forgiveness is calculated as the lesser of eight weeks' (8/52) of 2019 or 2020 compensation or $15,385 per individual in total across all businesses.

  The total amount of compensation of owners who work at their business that is eligible for forgiveness depends on the business type. If total compensation across businesses that receive a PPP loan exceeds the $20,833 cap, owners can choose how to allocate the capped amount across different businesses.

  - C Corporations: The employee cash compensation of a C-corporation owner-employee (an owner who is also an employee (including where the owner is the only employee)), is eligible for forgiveness up to the amount of 2.5/12 of his or her 2019 or 2020 employee cash compensation up to a $20,833 cap, with cash compensation defined as it is for all other employees to include cash compensation plus employer retirement and health, life, disability, vision, and dental insurance contributions made on their behalf (i.e., the same as for employees generally). Payments other than for cash compensation should be included on lines 6-8 of PPP Schedule A of SBA Form 2508 (or lender equivalent), for borrowers using that form, and do not count towards the $20,833 cap per individual.

  - S Corporations: The cash compensation of an S-corporation owner-employee (an owner who is also an employee) is eligible for loan

forgiveness up to the prorated amount of their 2019 or 2020 cash compensation up to a $20,833 cap, with cash compensation defined as cash compensation plus employer retirement contributions made on their behalf. Employer contributions for health, life, disability, vision, and dental insurance are not eligible for additional forgiveness for S-corporation employees with at least a 2% stake in the business, including for employees who are family members of an at least 2% owner under the family attribution rules of 26 U.S.C. 318, because those contributions are included in cash compensation. The eligible non-cash compensation payments should be included on lines 7 and 8 of PPP Schedule A of SBA Form 2508, for borrowers using that form, and do not count towards the $20,833 cap per individual.

- Self-employed Schedule C (or Schedule F) Filers: The compensation of self-employed Schedule C (or Schedule F) individuals, including sole proprietors, self-employed individuals, and independent contractors, that is eligible for loan forgiveness is limited to either the prorated amount of 2019 or 2020 net profits or gross income as reported (or to be reported) on IRS Form 1040 Schedule C (or F) line 31 or line 7, as applicable, subject to the $20,833 cap (see question 10 of "Paycheck Protection Program: How to Calculate Maximum Loan Amounts – By Business Type"), excluding any qualified sick leave equivalent amount for which a credit is claimed under section 7002 of the Families First Coronavirus Response Act (FFCRA) or qualified family leave equivalent amount for which a credit is claimed under section 7004 of FFCRA. Separate payments for health insurance, retirement, or state or local taxes are not eligible for additional loan forgiveness; health insurance and retirement expenses may not be added to the forgiveness amount. If the borrower did not submit its 2019 IRS Form 1040 Schedule C (or F) to the lender when the borrower initially applied for the loan, it must be included with the borrower's forgiveness application.

    - If a Schedule C (or F) filer elects to use gross income to calculate its loan amount on a First Draw PPP loan and the borrower reported more than $150,000 in gross income on the Schedule C (or F) that was used to calculate the borrower's loan amount, the borrower will not automatically be deemed to have made the statutorily required certification concerning the necessity of the loan request in good faith, and the borrower may be subject to review by the SBA of its necessity certification (i.e. the less than $2 million loan necessity safe harbor will not apply to these borrowers).

- General Partners: The compensation of general partners that is eligible for loan forgiveness is limited to the prorated amount of their 2019 or 2020 net earnings from self-employment that is subject to self-employment tax, which is computed from 2019 IRS Form 1065 Schedule K-1 box 14a (reduced by box 12 section 179 expense deduction, unreimbursed partnership expenses deducted on their IRS Form 1040 Schedule SE, and depletion claimed on oil and gas properties) multiplied by 0.9235, all subject to the $20,833 cap. Compensation is only eligible for loan forgiveness if the payments to partners are made during the covered period. Separate payments for health insurance, retirement, or state or local taxes are not

eligible for additional loan forgiveness. If the partnership did not submit its 2019 IRS Form 1065 K-1s when initially applying for the loan, it must be included with the partnership's forgiveness application.

- LLC Owners: LLC owners must follow the instructions that apply to how their business was organized for tax filing purposes in the reference year used to determine their loan amount.

- **Group Health Care Benefits** – Employer expenses for employee group health care benefits that are paid or incurred by the borrower during the covered period are payroll costs eligible for loan forgiveness. However, payroll costs do not include expenses for group health care benefits paid by employees (or beneficiaries of the plan) either pre-tax or after tax, such as the employee share of their health care premium. Forgiveness is not provided for expenses for group health benefits accelerated from periods outside the covered period.

  If a borrower has an insured group health plan, insurance premiums paid or incurred during the covered period qualify as "payroll costs," as long as the premiums are paid during the applicable period or by the next premium due date after the end of the applicable period.

- **Retirement Benefits** – Generally, employer contributions for employee retirement benefits that are paid or incurred by the borrower during the covered period qualify as "payroll costs" eligible for loan forgiveness. The employer contributions for retirement benefits included in the loan forgiveness amount as payroll costs cannot include any retirement contributions deducted from employees' pay or otherwise paid by employees. Forgiveness is not provided for employer contributions for retirement benefits accelerated from periods outside the covered period.

- **Eligible Non-payroll Costs** – An eligible non-payroll cost must be paid or incurred during the covered period and paid on or before the next regular billing date, even if the billing date is after the covered period. As explained in the Forgiveness Rules, eligible non-payroll costs include any amounts paid during the covered period (regardless of when incurred if incurred *prior to* the covered period) and costs incurred during the covered period even if paid following the covered period. Other than in the case of mortgage interest, which the Forgiveness Rules expressly state cannot be prepaid, there is no express exclusion from eligible non-payroll costs for prepayments of utility or rental expenses.

- **Deductible Expenses:** The IRS had held that expenses that gave rise to PPP loan forgiveness were not deductible. The CAA reverses this rule and permits taxpayers whose PPP loans are forgiven to deduct the expenses relating to their loans to the extent they would otherwise qualify as ordinary and necessary business expenses. This rule applies retroactively to the effective date of the CARES Act so that expenses paid using funds from PPP loans previously issued under the CARES Act are deductible regardless of when the loan was forgiven.

- **Reduction in Forgiveness Amount:** The loan amount eligible for forgiveness will be reduced (i) **first,** dollar-for-dollar by the amount of any salary cut for any employee employed by the borrower during the covered period that is in excess of 25% of such employee's total salary or wages for the most recent full quarter before the covered period

and either (A) did not receive annualized compensation of $100,000 or more in any pay period in 2019 or 2020 or (B) was not employed by the employer in 2019 or 2020; and (ii) **second**, proportionally for reductions in the average number of FTEs during the covered period compared to the average number of FTEs per month during a reference period selected by the borrower. The borrower can select one of the following reference periods: February 15, 2019 to June 30, 2019, January 1, 2020 to February 29, 2020, or, in the case of seasonal employers, average number of FTEs per month between February 15, 2019 to June 30, 2019; between January 1, 2020 and February 29, 2020; or any consecutive 12-week period between February 15, 2019 and February 15, 2020. Note, in the case of seasonal employers, if such seasonal employer elects to use a 12-week period between February 15, 2019 and February 15, 2020 to calculate its maximum PPP loan amount, the employer must use the same 12-week period as the reference period for calculation of any reduction in the amount of loan forgiveness. Form 3508 contains a worksheet that provides step-by-step instructions for calculating such reductions.

- **Reduction in Salary or Wages** – For purposes of calculating reductions in the loan forgiveness amount, the borrower should only take into account decreases in salaries or wages (not total compensation (*e.g.*, bonus reductions)).

- **FTE Reduction Exception** – As detailed in SBA Form 3508, no reductions are required for the following categories of employees and the borrower can include the FTE calculation of such employees in its calculation of average FTE for the covered period (as if such employee were still employed). Categories (1) and (2) below are expressly contemplated in the PPPFA.

    - (1) any positions for which the borrower made a good-faith, written offer to rehire an individual who was an employee on February 15, 2020 and the borrower was unable to hire similarly qualified employees for unfilled positions **on or before** (a) December 31, 2020, for a PPP loan made before December 27, 2020 or (b) the last day of the covered period, for a PPP loan made on or after December 27, 2020;

    - (2) any positions for which the borrower made a good-faith, written offer to restore any reduction in hours, at the same salary or wages, during the covered period and the employee rejected the offer; and

    - (3) any employee who during the covered period (a) was fired for cause, (b) voluntarily resigned, or (c) voluntarily requested and received a reduction of hours.

  A borrower cannot include the FTE calculation for such employees if the position was filled by a new employee (*i.e.*, borrower cannot double-count such former and replacement employee for the same position). *For example, if during the selected covered period a borrower fired for cause an employee with an average of 20 hours paid per week, the borrower can include 0.5 FTE in its average FTE calculations even though that employee is no longer employed. However, if the borrower filled the position of the fired employee with a new employee and that new employee has an average of 30 hours paid per week, the borrower can include only the 0.75 FTE for the new employee.* Further, while not required to be submitted with its application, the borrower must retain documentation supporting the applicability of these exceptions (*see* **Question 8** below).

The FTE Reduction Exceptions apply to **all employees**, including employees who made more than $100,000 in 2019 or 2020.

- **Average FTE** – Average FTE during the covered period is determined using the average number of hours paid per week, divided by 40, and rounded to the nearest tenth. This calculation is done on an employee-by-employee basis and the maximum FTE for each employee is capped at 1.0 (*for example: (i) if the average number of hours paid per week for an employee is 45, that employee counts as 1 FTE and (ii) if the average number of hours paid per work for an employee is 30, that employee would count as 0.75 FTE*). Borrowers can use a simplified method that assigns 1.0 for employees who work 40 hours or more per week and 0.5 for those who work fewer, but should note that doing so may understate FTEs if a borrower's employees are working less than 40 but more than 20 hours per week. New employees not employed during the reference period can be included in the calculation of average FTE for the covered period.

- **No Double Penalty for Salary Decline Due to FTE Reduction** – Under the Forgiveness Rules, to ensure that borrowers are not doubly penalized, the salary/wage reduction applies only to the portion of the decline in employee salary and wages that is *not* attributable to the FTE reduction. (*The SBA provides the following example*: "*An hourly wage employee had been working 40 hours per week during the borrower selected reference period (FTE of 1.0) and the borrower reduced the employee's hours to 20 hours per week during the covered period (FTE of 0.5). There was no change to the employee's hourly wage during the covered period. Because the hourly wage did not change, the reduction in the employee's total wages is entirely attributable to the FTE reduction and the borrower is not required to conduct a salary/wage reduction calculation for that employee*.")

- **Safe Harbors to Forgiveness Reduction:**

  - **Salary/Hourly Wage Reduction Safe Harbor** – The safe harbor for reductions in salary/wages of applicable employees must be assessed on an employee-by-employee basis.

    - **Pre-Economic Aid Act PPP loans** – A borrower who received a loan prior to the Economic Aid Act (December 27, 2020) is exempt from a reduction with respect to an employee if both: (1) the borrower reduced that employee's compensation by more than 25% in the period beginning February 15, 2020 and ending April 26, 2020; and (2) the *average* annual salary or hourly wages of that employee as of December 31, 2020 is equal to or greater than that employee's annual salary or hourly wages as of February 15, 2020.

      Form 3508 (PPP Schedule A Worksheet) indicates that this safe harbor applies if the reduction is restored as of the *earlier of* December 31, 2020 and the date that the forgiveness application is submitted. Therefore, borrowers were not required to wait until year-end to restore compensation and submit a forgiveness application. Borrowers could restore compensation levels at an earlier time, and, once restored, utilize the safe

harbor in a forgiveness application. It remains, however, an open question as to how long such restored compensation must be preserved.

- **Post-Economic Aid Act PPP loans** – For a borrower who received a loan after the enactment of the Economic Aid Act, such loan is exempt from a reduction with respect to an employee if both: (1) the borrower reduced that employee's compensation by more than 25% in the period beginning February 15, 2020 and ending April 26, 2020; and (2) the *average* annual salary or hourly wages of that employee as of the last day of the covered period for such loan is equal to or greater than that employee's annual salary or hourly wages as of February 15, 2020.

- **What does "average" mean? –** The implication of "average" in this context is unclear. Is it sufficient for compensation to be restored by December 31, 2020 to the same annualized salary amount or hourly wages that an employee was receiving on February 15, 2020 (*for example, if an* employee *was making $5,000 per month ($60,000 annualized salary) as of February 15 and is reduced to $3,000 per month on March 1, does that employee simply need to be restored to $5,000 per month going-forward as of December 31*)? Does "average" imply that an employee needs to be "caught up" so the average salary or hourly wages for year-to-date as of December 31, 2020 (or for post-Economic Aid Act PPP loans, the end of the covered period for a PPP loan) is equal to or greater than annual salary or hourly wages as of February 15, 2020 (*for example, would the employee need to receive $8,000 to make up for $2,000 less in monthly compensation for March – June, so that the employee's average annualized salary as of December 31 is the same as on February 15 ($60,000))?* This is a critical question, as being required to deliver make-up payments will likely prove untenable for many employers. In our experience, the general practice to date has been to assume that the salary or the hourly wages of a given employee as of the last day of the covered period needs to be equal to or greater than that employee's salary or hourly wages as of February 15, 2020 to qualify for this safe harbor, reading out the "average."

- **Employee Availability FTE Reduction Safe Harbor (FTE Reduction Safe Harbor 1 in Form 3508)** –

  - **Pre-Economic Aid Act PPP loans** – The PPPFA added an exemption (that applies during the period beginning February 15, 2020 and ending December 31, 2020) from the reduction in loan forgiveness for reduction of the number of FTEs if a borrower in good faith is able to document an inability to return to the same level of business activity as such business was operating before February 15, 2020 due to compliance with requirements established or guidance issued by the Secretary of Health and Human Services, the director of the CDC, or OSHA (or any state or local government shutdown orders issued pursuant to such guidance) during the period beginning on March 31, 2020 and ending on December 31, 2020, related to the maintenance of standards for sanitation, social distancing, or any other worker or customer safety requirements related to COVID-19.

This safe harbor is captured in the Interim Final Rule on loan forgiveness published on June 22, 2020, and protects a borrower broadly from any reductions in FTE levels between February 15, 2020 and the end of the covered period due to compliance with requirements/guidance of the above federal departments (or any state or local government shutdown orders issued in compliance with such federal requirements or guidance). The borrower must expressly certify as to the applicability of this safe harbor. Important questions remain with respect to this safe harbor:

- Does this safe harbor provide complete coverage from an FTE-based reduction to the forgiveness amount regardless of the duration or the extent of the reduction in the level of business activity? For example, if a borrower was required to operate at half capacity for only 3 weeks of the period between February 15, 2020 and the end of the covered period, does this safe harbor nonetheless provide complete protection? The answer appears to be yes and in our experience this has been the general practice to date.

- How is "same level of business activity" defined? Is this purely a measure of physical operations (*e.g.*, a business was required to close, operate at less than full capacity, or provide curb-side pickup/delivery/take-away services), or can it be measured in terms of economic reductions (*e.g.*, if a business remained completely open, but due to COVID-19 and related federal guidelines/requirements suffered a decline in demand for its services)?

- **Post-Economic Aid Act PPP loans** – Form 3508 has been updated to reflect the changes under the Economic Aid Act, and the exemption period for a PPP loan made after December 27, 2020 applies from March 1, 2020 until the last date of the covered period for any such loan.

- **FTE Reduction Safe Harbor (FTE Reduction Safe Harbor 2 in Form 3508)** –

  - **Pre-Economic Aid Act PPP loans** – Borrower is exempt from the reduction in loan forgiveness for reduction of the number of FTE employees if both of the following conditions are met: (1) the borrower **reduced** its average FTE levels in the period beginning February 15, 2020 and ending April 26, 2020; and (2) the borrower restored, by **not later than December 31, 2020**, its total FTE levels to its total FTE levels for the pay period inclusive of February 15, 2020. Borrower is instructed to calculate FTE for each relevant period (February 15 to April 26, 2020, the pay period inclusive of February 15, 2020, and total FTE as of December 31, 2020) using the same calculation methods required for determining average FTE during the covered period (described above).

    Form 3508 (PPP Schedule A Worksheet) indicates that FTE Reduction Safe Harbor 2 applies if the reduction is restored as of the earlier of December 31, 2020 and the date that the forgiveness application is submitted. Borrowers were not required to wait until year-end to restore

FTE levels and submit a forgiveness application. Borrowers could restore FTE levels at an earlier time, and, once restored, utilize this safe harbor in a forgiveness application. It remains, however, an open question as to how long such restored FTE levels must be preserved. Note that, as written, the condition in Form 3508 and the Forgiveness Rules that the borrower must have reduced average FTE employees during the February 15 to April 26, 2020 period seems to imply that so long as the borrower had any reduction during that period *all reductions* (whether during that period or after) could have been cured by December 31, 2020. This conflicts with prior guidance that suggested that reductions occurring *after* April 26, 2020 were incurable.

  - **Post-Economic Aid Act PPP loans** – Under the Economic Aid Act, the restoration must instead occur by the end of the covered period for a PPP loan made after December 27, rather than December 31, 2020, and Form 3508 has been updated accordingly.

- **All or Nothing Test** – The FTE reduction safe harbor appears to be an "all or nothing" test and any partial restoration in total FTE as of December 31, 2020 for pre-Economic Aid Act PPP loans and the end of the applicable covered period for post-Economic Aid Act PPP loans below the total FTE for the pay period inclusive of February 15, 2020 is insufficient for the safe harbor.

- **Simplified Option** – Form 3508 (PPP Schedule A Worksheet) provides borrowers a simplified option to determine if an FTE-based reduction in the forgiveness amount is required. If a borrower "has not reduced the number of [its] employees or the average paid hours of [its] employees between January 1, 2020 and the end of the Covered Period," then the borrower is not subject to an FTE-based reduction. While not a "safe harbor," this provision is effectively a short-cut around the morass of calculating a borrower's FTE-based reduction to the forgiveness amount. Note however that it is unclear if this option is conditioned on (i) no reductions *on average* between January 1, 2020 and the end of the covered period as compared to the numbers of employees or average paid hours as of January 1, 2020 (such that reductions restored during the covered period) would not affect the availability of this safe harbor), or (ii) no reductions <u>at all</u>, at any time during such period (even if restored).

- **Open Questions on FTE Reductions:** Some important questions remain open, including:

  - If a borrower restored or restores employee compensation or FTE levels prior to the end of the applicable period (whether December 31, 2020 or the end of the covered period for the applicable PPP loan) and submits a forgiveness application availing itself of the applicable safe harbor, how long does such restored compensation/FTE level need to be preserved? There is no guidance related to if and what a borrower is required to do after December 31, 2020 or the end of the covered period. Would there be any consequences to a borrower reducing its FTE count and/or reducing compensation to employees at that point in time? Form 3508 does require the borrower to provide the number of its employees as of the date it applied for the loan and as of the date it applied for forgiveness. Is that meaningful

in any way? Given that the loan forgiveness process is likely to last several months after the end of the covered period, this could be an important issue.

- It remains unclear how a fully-furloughed employee who is not receiving compensation, but continues to receive benefits from the borrower, is to be treated for purposes of calculating (and reducing) the forgiveness amount. While the Forgiveness Rules indicate that a reduction in an employee's wages/salary that is the result of a reduction in hours does not create a "double penalty" for purposes of reducing the forgiveness amount (such that only the FTE-based reduction applies), what about a fully-furloughed employee who has had their hours eliminated (and as a result their compensation reduced to $0)? Is it indeed the case that if a borrower was forced to fully furlough 50% of its workforce, in part to ensure sufficient funds to continue to pay for the healthcare benefits for such employees, that the borrower may suffer a 50% reduction in its forgiveness amount?

- **Forgiveness Application Review Process:**
  - **Application Review Flag:** Borrowers that, together with their affiliates, received PPP loans in excess of $2 million are required to check a box on Form 3508 to so indicate. This will be used to flag applications required to be reviewed by the SBA.
  - **Certifications and Materials:** Borrowers must certify (among other certifications) that the dollar amount for which forgiveness is requested (i) was used only for eligible expenses, (ii) has been appropriately reduced (for compensation or average FTE reductions), (iii) includes payroll costs equal to at least 60% of the forgiveness amounts, and (iv) for any owner-employee or self-employed individual/general partner, does not exceed **2.5 months** of compensation for the year used to calculate the PPP loan amount, capped at $20,833 in total per individual across all businesses. The application also reinforces that there are potential criminal charges for false claims in connection with the information provided in the application or supporting documents or if funds were knowingly used for unauthorized purposes. Form 3508 includes a fulsome list of materials that a borrower must submit and/or prepare and maintain with respect to its application for forgiveness (see **Question 8** below), and makes clear that the borrower must retain all such materials for 6 years and provide SBA authorized representatives access upon request.
  - **Forgiveness Application Timing:** To receive loan forgiveness, a borrower must submit its Form 3508 together with the other materials required under Form 3508 or requested by the lender (see **Question 8** below) **within 10 months** of the completion of the covered period. The Interim Final Rule on loan forgiveness, published on June 22, 2020, clarifies that a borrower may submit a loan forgiveness application any time on or before the maturity date of the loan, ***including before the end of the covered period***, if the borrower has used all of the loan proceeds for which the borrower is requesting forgiveness. As affirmed in the Loan Forgiveness FAQs, so long as a borrower submits its loan forgiveness application within that time frame, the borrower is not required to make any payments until the forgiveness amount is remitted to the lender by the SBA. If the loan is fully forgiven, the borrower is not responsible for any payments. If only a portion of the loan is forgiven, or if the forgiveness application is denied, any remaining balance due on the loan must be repaid by the borrower on or before the maturity date of the loan.

- **Forgiveness Application and Review for PPP loans of up to $150,000:** Loans that are not more than $150,000 may be forgiven if the PPP borrower submits a one (1) page certification, Form 3508S, which, consistent with the requirements of the Economic Aid Act, contains: (i) the number of employees the eligible recipient was able to retain because of the covered loan, (ii) the estimated amount spent on payroll costs, and (iii) the total loan value. A borrower with a loan of $50,000 or less, other than any borrower that together with its affiliates received (x) First Draw PPP Loans totaling $2 million or more or (y) Second Draw PPP Loans totaling $2 million or more, has the additional benefit of being exempt from any reduction in the borrower's loan forgiveness amount based on reduction in FTE employees or reductions in employee salary or wages that would otherwise apply. By completing such form, the PPP borrower attests that the information provided is accurate and that it complied with the requirements under section 7(a)(36). Such PPP borrower is required to retain records that affirm compliance with such requirements (as to employment records, for the four (4) year period following submission, and for other records, for the three (3) year period following submission). The Economic Aid Act expressly states that such PPP borrower is not required to submit as part of the forgiveness application process any additional application or documentation to substantiate forgiveness. This new easy application process for loans of not more than $150,000 has retroactive effect, and applies to existing PPP loans and PPP loans made on or after the enactment of the Economic Aid Act. The SBA can review and audit such loans and access any records the borrower is required to retain.

- **Forgiveness Review for PPP loans in excess of $150,000:** The Economic Aid Act does not alter the existing rules and processes for reviewing PPP loans with a principal balance in excess of $150,000.

    - The Review Rules (as currently in effect) require the lender to confirm receipt of all requisite forgiveness documentation and to use such materials to confirm certain of the borrower's calculations as part of a "good-faith review." The lender may rely on borrower representations/certifications and the onus remains on the borrower to provide an accurate calculation of the loan forgiveness amount and to supply accurate information and calculations in its forgiveness application.[xv] If lenders identify errors in a borrower's calculation or material lack of substantiation in the supporting documents, lenders are directed to work with the borrower to remedy the issue (*i.e.*, as opposed to denying forgiveness without an opportunity to ameliorate such deficiencies).

    - The lender must make a determination as to loan forgiveness not less than 60 days from receipt of a "complete application" and report its decision to the SBA. The lender must also notify the borrower of its decision.

    - **Lender Confirmation Required for PPP Forgiveness Submissions** – The lender must confirm the following for each PPP forgiveness submission before the SBA will accept the submission:

        - (1) the submission accurately reflects the lender's decision regarding the borrower's loan forgiveness application;

        - (2) the information provided by the lender to the SBA with the submission accurately reflects the lender's records for the PPP loan;

- (3) the lender has made its decision in accordance with the requirements set forth in Part III.2.a. of the PPP Interim Final Rule on SBA Loan Review Procedures and Related Borrower and Lender Responsibilities, as amended;

- (4) the PPP loan has not been cancelled or repaid; and

- (5) the lender has not issued a previous loan forgiveness decision to the SBA for this PPP loan, unless it is a resubmission following a rejection or a reconsideration of a denial without prejudice.

- **Lender Approves Forgiveness (All or Part)** – If a lender determines a borrower is entitled to forgiveness of all or a portion of the amount requested, the lender must request payment from the SBA when it delivers its forgiveness determination to the SBA.[xvi] The SBA will, not later than 90 days after the lender issues its decision to the SBA, remit the forgiveness amount to the lender, plus any interest accrued through the date of payment (previously, EIDL COVID-19 advances would have reduced this amount, but such a reduction was eliminated under the Economic Aid Act). This timeframe is subject to any SBA review of the loan/loan application, during which time a loan may not be forgiven (the SBA Review Process is discussed further below). The forgiveness process may take as many as 150 days assuming no issues that create delays (e.g., SBA/lender information requests, SBA undertakes a review of the loan/loan application).

  Per the Review Rules, if the amount remitted by the SBA exceeds the remaining principal balance because the borrower made scheduled payments on the loan after the payment deferral date, the lender must pay the excess amount (including accrued interest) to the borrower.

- **Lender Denies Forgiveness (All or Part**) – If a lender issues its decision to the SBA that all or a portion of the requested forgiveness amount is to be denied, the lender must (i) provide the SBA a reason for such denial, and (ii) notify the borrower of such decision. The SBA has the right to review such determination in its sole discretion. Within 30 days of notice from the lender, a borrower may request that the SBA review the lender's decision. While it appears that such review must be accomplished within the 90 day forgiveness-review period, the Review Rules are not clear and such review may exceed that time frame. If only a portion of the loan is forgiven or if the forgiveness request is denied, the balance must be repaid by the borrower on or before the 2-year maturity of the loan.

- **Reduction for EIDL Advances** – The CARES Act required that an EIDL advance ($10,000) be deducted from a PPP loan borrower's forgiveness amount. The Economic Aid Act removes this requirement, and it has retroactive effect to the enactment of the CARES Act.

- **PPP Forgiveness Platform** – Treasury and the SBA have published guidance on the procedures for lender submission of PPP loan forgiveness decisions to the SBA and SBA loan forgiveness reviews, which is available for **download**. The SBA has partnered with a financial services technology provider – Goldschmitt-CRI – to make available a secure SaaS platform (the "PPP Forgiveness Platform") to accept loan forgiveness decisions, supporting documentation, and requests for

forgiveness payments. The PPP Forgiveness Platform is available only to PPP lenders, not PPP borrowers. The PPP Forgiveness Platform makes available a user interface for lenders to upload required data and documentation, monitor the status of the forgiveness request, and respond to the SBA in case of an inquiry or if the SBA selects the loan for review. Lender submissions may be rejected by an initial screening process in the PPP Forgiveness Platform, including if they are incomplete or contain errors. If a lender submission is rejected, the lender will be notified by the PPP Forgiveness Platform. The lender must then correct the submission and resubmit it to the SBA. Lender correction of a submission will restart the 90-day period for the SBA to remit the payment.

- **Forgiveness Audit Plan:** The Economic Aid Act requires the SBA to present to Congress by February 10, 2021 (45 days after the December 27, 2020 enactment of the Economic Aid Act) an audit plan that details (i) policies and procedures that the SBA intends to use for conducting forgiveness reviews and audits of PPP loans and the metrics that will be used to determine which loans to audit. Within 30 days after the submission of such audit plan (and on a monthly basis thereafter), the SBA must submit to Congress a report on the forgiveness review and audit activities conducted that will include (i) the number of active reviews and audits, (ii) the number of reviews and audits that have been ongoing for more than 60 days, and (iii) any substantial changes made to the audit plan.

- **PPP in the Context of M&A Transactions:**

  On October 2, 2020, the SBA published a procedural notice (Control No. 5000-20057) regarding PPP loans and changes of ownership (the "Notice"). The Notice describes the circumstances under which notice and consent of the PPP lender and the SBA may be required in connection with a "change of ownership" of a PPP borrower. The Notice further details the actions that the parties to a change of ownership transaction can take to negate the requirement of SBA approval.

  - **What constitutes a Change of Ownership:** For purposes of the PPP, a "change of ownership" will be considered to have occurred when (1) at least 20% of the common stock or other ownership interests of a PPP borrower (including a publicly traded entity) is sold or otherwise transferred, whether in one or more transactions, including to an affiliate or an existing owner of the entity, (2) the PPP borrower sells or otherwise transfers at least 50% of its assets (measured by fair market value), whether in one or more transactions, or (3) a PPP borrower is merged with or into another entity. Note that the guidance appears to apply only to a change in ownership of the PPP borrower itself and not to transactions in any parent entity.

  - **Aggregation of Post-Approval Transactions:** All sales or transfers occurring following the approval date of the PPP loan (*i.e.* the operative date, not the disbursement date) are aggregated to determine whether a change of ownership has occurred or is occurring.

  - **Publicly Traded Borrowers:** The applicability of the aggregation construct is somewhat limited in the case of publicly traded PPP borrowers. Only sales/transfers that result in one person or entity holding or owning at least 20% of the common stock or other ownership interest of the PPP borrowers are aggregated. As noted above, under the Economic Aid Act, public companies will not be eligible for PPP loans going forward.

- **No Approval Required:** The new guidance means that transfers of less than 20% of common stock or ownership interests or less than 50% of the assets of a borrower do not require SBA approval. These transactions may or may not require the PPP lender's approval, depending on the loan documentation.

- **Retention of Responsibility:** Notwithstanding a change of ownership, the PPP borrower remains responsible for: (1) performance of all obligations under the PPP loan; (2) certifications made in connection with the PPP loan application, including economic necessity; (3) compliance with all other applicable PPP requirements; and (4) obtaining, preparing, and retaining all required PPP forms and supporting documentation and providing such forms/documentation to the PPP lender or the SBA upon request. A change of ownership does not impact the SBA's rights and remedies in the case of fraud, false statements, and unauthorized uses of PPP loans with respect to the PPP borrower.[xvii]

- **Notices to the PPP Lender:** While most PPP loan Notes already provide that a PPP lender[xviii] must be notified and its consent obtained prior to the closing of a change of ownership, the Notice affirms that in all instances (which appears to cover instances where a PPP loan Note may be silent as to Lender consent/notices) the PPP borrower must deliver written notice to the PPP lender prior to the closing of a contemplated change of ownership transaction. That notice must include the agreements/documents that effectuate the transaction (*i.e.*, equity purchase agreement, merger agreement, asset purchase agreement, etc.). The language of the Notice suggests that the PPP lender must receive a notice regardless of whether there are restrictions on the change of ownership (discussed below).

- **Not Subject to Restrictions if PPP Note is Fully Satisfied at Closing:** The Notice provides that there are no restrictions on a change of ownership if, prior to the closing, the PPP borrower has either:

    - (a) repaid the PPP Note in full (which would seem to encompass, e.g., a payoff at the closing of an acquisition transaction as is typical for target indebtedness), or

    - (b) completed the loan forgiveness process and (i) the SBA has remitted funds to the PPP lender in full satisfaction of the PPP Note; or (ii) the PPP borrower has repaid any remaining PPP loan balance.

- **Restrictions Apply if the PPP Note is Not Fully Satisfied at Closing:** A change of ownership of a PPP borrower is subject to restrictions when it is not fully satisfied (either through payoff or forgiveness) prior to closing of the proposed transaction. Where the PPP loan remains outstanding as of closing and a change in ownership occurs, (i) in certain instances only the PPP lender's consent will be required and (ii) in other instances the consent of both the lender *and* the SBA is required.

- **When is the SBA Prior Approval Not Required:** The Notice states that the prior approval of the SBA will not be required if the following conditions are met, in which case the PPP lender alone may approve the change of ownership:

    - **Equity Sale/Merger –** Common stock or other ownership interest in a PPP borrower may be sold/transferred without SBA approval if:

- (a) **50% or less Transfer/Sale –** The sale/transfer is of 50% or less of the common stock or other ownership interest of the PPP borrower (for which purpose all post approval date transactions are aggregated); or

- (b) **Completed Forgiveness Application and Escrowed Funds** – (i) The PPP borrower completes a forgiveness application reflecting its use of all of the PPP loan proceeds and submits it (with required supporting documents) to the PPP lender, and (ii) an amount equal to the outstanding balance of the PPP loan is deposited into an interest-bearing escrow account that is controlled by the PPP lender. Escrowed funds will be disbursed following the forgiveness process (including any appeal of an SBA decision) to first satisfy any remaining PPP loan balance (principal + interest) and then as directed by the transaction parties.

- **Asset Sale** – 50% or more of a PPP borrower's assets (measured by fair market value) may be sold without SBA approval if the forgiveness application is completed and escrow established (as described above, including with respect to the release of escrowed funds).[xix]

- **When is the SBA Prior Approval Required:** If the conditions described above are not satisfied, then the PPP lender cannot unilaterally approve a change of ownership and the SBA's prior approval is also required.

  - **SBA Approval Request –** To obtain such SBA approval, the PPP lender must submit a request to the SBA. Note that it is the PPP lender and not the PPP borrower that must submit this request, which may have additional ramifications on transaction timing (e.g., as any follow-up SBA request would first have to be reviewed and relayed by the PPP lender). Such a request must include the following: (i) the reason that the PPP borrower cannot fully satisfy (i.e., repay) the PPP Note or escrow funds (as described above); (ii) the details of the requested transaction; (iii) a copy of the executed PPP Note; (iv) any LOI and the purchase or sale agreement setting forth the responsibilities of the PPP borrower, seller (if different from the PPP borrower), and buyer; (v) disclosure of whether the buyer has an existing PPP loan (and if so, the SBA loan number); and (vi) identification of all owners of 20% or more of the purchasing entity.

  - **Additional Risk Mitigation Measures** – The SBA (as it deems appropriate) may require "additional risk mitigation measures" as a condition to its approval. This broad language renders it difficult to ascertain what such additional mitigation measures may entail.

  - **Asset Sales** – The SBA's approval of a sale of 50% or more of a PPP borrower's assets (measured by fair market value) will be conditioned on the purchasing entity assuming all of the PPP borrower's obligations under the PPP loan (including responsibility for compliance with the PPP loan terms), which assumption must be explicitly addressed in the transaction agreement or in a separate assumption agreement that is submitted to the SBA.

  - **Timing** – The SBA will provide a determination within 60 days of its receipt of a complete request.

- **Additional Rules for ALL Sales/Transfers of Common Stock or Other Ownership Interests or Mergers:** For all equity sales/transfers or mergers constituting a change of ownership, whether or not the SBA's prior approval is required (as described above), the Notice imposes the following additional obligations:

  - **Retention of Obligations –** Without limiting the general language (described above) regarding retention of obligations, the Notice expressly provides that the PPP borrower (or if the PPP borrower is not the surviving entity in merger, the successor to the PPP borrower) will remain subject to all obligations under the PPP loan.

  - **Recourse Against Owners for Unauthorized Use** – The SBA will have (direct) recourse against any new owner(s) that use PPP funds for unauthorized purposes.

  - **Responsibilities if Owner(s)/Successor Have a PPP loan** – If any of the new owner(s) or the successor arising from such a transaction has a separate PPP loan, then, following the closing: (i) in the case of an equity sale/transfer, the PPP borrower and the new owner(s) are responsible for segregating and delineating PPP funds and expenses and providing documentation to demonstrate compliance with PPP requirements by each PPP borrower, and (ii) in the case of a merger, the successor is responsible for segregating and delineating PPP funds and expenses and providing documentation to demonstrate compliance with PPP requirements with respect to both PPP loans.

  - **PPP Lender Reporting Obligations –** Within 5 business days of closing, the PPP lender must notify the SBA (to the appropriate SBA Loan Servicing Center) of (i) the identity of the new owner(s); (ii) the new owner(s) ownership percentage(s); (iii) the TIN(s) of any owner(s) holding 20% or more of the equity in the business; and (iv) the location and size of any escrow account under the control of the PPP lender.

- **Acquisition Does NOT Cause Forfeiture of Employee Retention Tax Credit:** Whereas the CARES Act denied the Employee Retention Tax Credit ("ERTC") to any employer that receives a PPP loan, and defined the term "employer" expansively, potentially causing acquiring corporations with ERTCs to lose or recapture those tax credits if they acquired a target company that had received a PPP loan, the CAA instead permits an employer that receives a PPP loan, whether by acquisition of a company with a PPP loan or direct application for a PPP loan, to receive an ERTC. This change applies retroactively to the effective date of the CARES Act. For additional information on the tax provisions contained in the CAA more broadly please see our *Tax Talks* blog post *Coronavirus: President Trump Signs Consolidated Appropriations Act, 2021; Summary of the Tax Provisions*.

- **SBA Review Process:** Under the Review Rules, the SBA has broad authority to review any PPP loan at any time, in its discretion (including after a loan is forgiven).

  - **Scope of Review:** The SBA may review: (i) borrower eligibility (based on the CARES Act, rules and guidance in effect at the time of its application, and its Form 2483), including the application of the SBA's affiliation rules (see Size Standard and Affiliation above) and list of ineligible industries (as modified for the PPP) (see Ineligible Industries below); (ii) loan amount calculation and use of proceeds; and (iii) loan forgiveness amount claimed by the borrower. If the SBA undertakes a review of a PPP loan, it will notify the lender

and the lender must notify the borrower in writing within 5 business days. A lender cannot approve any application for loan forgiveness until the SBA notifies the lender in writing that it has completed its review.

- **SBA Requests:** If loan documentation submitted to the SBA or any other information indicates that a borrower may be ineligible for a PPP loan or may be ineligible to receive the loan amount or loan forgiveness amount claimed by the borrower, the SBA will (directly or via the lender) request additional information from the borrower and the SBA will consider all information provided in response. Failure to respond may result in a finding of ineligibility or that a borrower is ineligible for the loan amount/forgiveness amount claimed.

- **SBA Determinations:** If the SBA determines that a borrower was ineligible for the PPP loan (*e.g.*, because the borrower lacked an adequate basis for the certifications made in its PPP loan application) the loan will not be eligible for loan forgiveness and the SBA will direct the lender to deny the forgiveness application. If the SBA determines that the borrower is ineligible for the loan amount or forgiveness amount claimed, the SBA will direct the lender to deny the loan forgiveness application in whole or in part, as applicable. Such denial may be in addition to the SBA's exercise of other remedies (including, *e.g.*, repayment of the PPP loan) and may expose the borrower to penalties (discussed below under "Consequences of a False Filing")

- **Appeal of Final SBA Loan Review Decisions:** An Interim Final Rule published on August 11, 2020 contains a largely technical set of rules governing how a borrower can appeal to the SBA's Office of Hearings and Appeals (the "OHA")[xx] certain final "SBA Loan Review Decisions." SBA Loan Review Decisions are limited to a final written finding by the SBA that a borrower (1) was ineligible for a PPP loan; (2) was ineligible for the PPP loan amount received or used the PPP loan proceeds for unauthorized uses; (3) is ineligible for PPP loan forgiveness in the amount determined by the lender in its full or partial approval decision issued to SBA; and/or (4) is ineligible for PPP loan forgiveness in any amount when the lender has issued a full denial decision to SBA. Only final SBA Loan Review Decisions can be appealed to the OHA, and only the borrower itself has standing to bring an OHA appeal (individual owners and lenders do not have such standing). An appeal petition must be filed within 30 calendar days after (i) the borrower's receipt of the final SBA Loan Review Decisions, or (ii) notification by the lender of the final SBA Loan Review Decisions, whichever is earlier. In bringing any such appeal, the borrower has the burden of proof, by a preponderance of the evidence, to establish that the SBA Loan Review Decision was based on a clear error of fact or law. A prevailing appellant is not entitled to recover attorney's fees.

  To be clear, a borrower cannot file an OHA appeal of a decision made by a lender concerning a PPP loan. Such a borrower must first request an SBA review of a lender decision (in accordance with the Interim Final Rule on SBA Loan Review, as amended). If the SBA renders a decision that constitutes an SBA Loan Review Decision, then it appears such a decision would be subject to an OHA appeal. It is notable that an appeal by a PPP borrower of any SBA Loan Review Decision does not extend the deferral period of the PPP loan.[xxi] An appeal to OHA is an administrative remedy that must be exhausted before judicial review of a final SBA Loan Review Decision may be sought in a federal district court.

- **Credit Elsewhere:** The SBA has waived the requirement that a borrower not be able to obtain financing elsewhere (but see discussion of the "necessity" certification above).

- **Disbursements**: Lenders must make a one-time, full disbursement of a PPP loan within 10 calendar days of approval (the date on which the SBA assigns a loan number). Loans that have not been disbursed because a borrower fails to submit required loan documentation within 20 days of loan approval are cancelled.

- **Other Economic Considerations:** PPP loans are non-recourse obligations provided that the loan proceeds are used for permitted purposes. No yearly or guarantee SBA fees will be charged.

- **Lender Fee Limits:** Processing fees paid to lenders will be based on the balance of the loan outstanding at the time of final disbursement.

    - **Pre-Economic Aid Act Loans** – For PPP loans made prior to December 27, 2020, a lender will receive a fee equal to a percentage of such final disbursement as follows: (i) 5.00% for loans of not more than $350,000; (ii) 3.00% for loans of more than $350,000 and less than $2 million; and (iii) 1.00% for loans of not less than $2 million. Lenders may not collect any fees from the applicant.

    - **Post-Economic Aid Act Loans** -- PPP loans made on or after December 27, 2020 are subject to the following adjusted fee scale for lender processing reimbursement: (i) the lesser of $2,500 and 50 percent of the balance of the financing outstanding at the time of disbursement, for loans of not more than $50,000, (ii) 5.00% for loans of more than $50,000 and not more than $350,000; (iii) 3.00% for loans of more than $350,000 and less than $2.0 million; and (iv) 1.00% for loans of not less than $2 million.

  The Review Rules provide that if the SBA conducts a review of a PPP loan and determines that a borrower is ineligible, then the lender is not eligible for a processing fee. Lender fees are subject to claw-back within 1 year of disbursement of a PPP loan if the SBA determines that a borrower was ineligible. However, if the lender has received a processing fee on a loan that was cancelled or voluntarily terminated and repaid after disbursement (including if a borrower repaid the PPP loan proceeds to conform to the borrower's certification regarding the necessity of the PPP loan request), the SBA will not require the lender to repay the processing fee unless the lender is found guilty of an act of fraud in connection with the PPP loan.

- **Agent Fee Limits:** The CARES Act authorizes the SBA to establish limits on fees that can be collected by agents that assist applicants in applying for the PPP. The PPP Rules provide that the fees of such agents will be paid by the lender out of the fees the lender receives from the SBA (*i.e.,* the agent may not collect fees from the borrower or be paid out of PPP loan proceeds). The Economic Aid Act clarifies that PPP loan recipients may not pay agents retained to prepare applications for a covered loan with covered loan proceeds and lenders are only permitted to pay fees for which the lender directly contracts the agent. Further, lenders are not required to repay reimbursement for loans unless found guilty of fraud in connection with the covered loan. The total amount an agent can collect from a lender for providing such assistance is capped at: (i) 1.00% for loans of not more than $350,000

(≤$3,500); (ii) 0.50% for loans of more than $350,000 and less than $2 million ($1,750 - ~$9,999); and (iii) 0.25% for loans of at least $2 million ($5,000+).

- **Application:** Each applicant seeking a 7(a) loan under the PPP is required to submit a Paycheck Protection Program Borrower Application Form (SBA Form 2483) (or SBA Form 2483-C for Schedule C filers using gross income) to a participating lender (together with any other documentation required by the lender as part of the application process (see **Question 4** below)).

    - The SBA published guidance effective as of January 6, 2021, addressing potential barriers to accessing capital for minority, underserved, veteran, and women-owned businesses to ensure equitable access to Second Draw Loans. Most notably, the SBA announced that it will (i) accept PPP loan applications only from community financial institutions for at least the first two days when the PPP loan portal re-opens on January 11, 2021, (ii) direct Lender Match (the SBA's free online tool to connect potential borrowers with SBA-approved lenders) borrower inquiries to small lenders who can aid traditionally underserved communities, and (iii) match small business through Lender Match with Certified Development Companies (non-profit organizations, each with a specific regional focus, that are certified by the SBA to administer SBA loans), Farm Credit System lenders (a nationwide lending network of financial institutions that provide credit to the agricultural community), microloan intermediaries, and traditional smaller asset size lenders.

- **Burden of Assessing Eligibility/Certifications:** PPP Rules and related SBA guidance place the burden on borrowers to confirm their own eligibility (including calculating payroll costs, assessing affiliation, and determining employee headcount) and the accuracy of the information they supply to lenders, permit lenders to rely on borrower certifications in determining loan eligibility, and provide that the SBA will hold lenders harmless for a borrower's failure to comply with PPP criteria.

- **Consequences of a False Filing:** An applicant is required as part of both Form 2483 (Loan Application) and Form 3508 (Forgiveness Application) to certify that it understands that knowingly making a false statement in order to obtain an SBA-guaranteed loan is punishable by law (including by imprisonment and significant monetary fines). Penalties include:

    - **Criminal Penalties** – Potential criminal penalties for false statements or fraud in connection with a PPP loan include (i) imprisonment of not more than 5 years and/or a fine of up to $250,000 (18 USC §§ 1001 & 3571); (ii) imprisonment of not more than 2 years and/or a fine of not more than $5,000 (15 USC § 645(a)); and (iii) imprisonment of not more than 30 years and/or a fine of not more than $1 million (18 USC § 1014).[xxii] Beyond the penalties expressly referenced in the PPP loan application, criminal penalties under other federal fraud statutes or SBA-specific criminal statutes (e.g., regarding embezzlement or concealment) may apply. For further discussion on the subject of potential criminal risks see our client alert Rear View Mirror: Criminal Exposure for Companies that Received PPP loans under the CARES Act.

    - **Civil Penalties** – In addition to criminal penalties, the government can pursue civil fraud remedies under the civil False Claims Act (31 U.S.C. 3729-3733) or the Program Fraud Civil Remedies Act (31 U.S.C. 3801-3812).

The threat of enforcement of such penalties is bolstered by the answer to Question 39 of the SBA's FAQs (published on April 29, 2020) and reiterated in the answer to Question 46 (published May 13, 2020) which state that the SBA "will review all loans in excess of $2 million, *in addition to other loans as appropriate*, following the lender's submission of the borrower's loan forgiveness application" (emphasis added). Given the potential risks and heightened scrutiny from Treasury, the SBA, the U.S. Justice Department (nationally and regionally), and the public and press more broadly of the companies receiving PPP loans, it is imperative the applicants carefully read and consider all certifications being made in Form 2483, Form 3508, and in any other documentation submitted to the SBA or a PPP lender.

II. Key Terms of the Second Draw Program

Section 7(a) of the Small Business Act is amended to add a new section (37) for the Second Draw Program. The parameters and terms governing the administration, permitted uses, and forgivable uses of the PPP and PPP loans are largely also applicable to the Second Draw Program and Second Draw Loans. On January 6, 2021, the SBA announced the Second Draw Rules, which confirm that the majority of the terms of the PPP Rules, as well as the FAQs and other guidance about PPP loans under section 7(a)(36) of the Small Business Act, also apply to the Second Draw Program. Below are terms and conditions specific to the Second Draw Program.

- **Eligibility:** Under the Second Draw Program, the parameters for borrower eligibility are narrower than under the PPP. To be eligible for a Second Draw Loan, an applicant must be a business concern, non-profit organization eligible for a First Draw PPP loan, housing cooperative, veterans organization, Tribal business concern, eligible self-employed individual, sole proprietor, independent contractor, small agricultural cooperative, eligible 501(c)(6) organization or destination marketing organization, an additional covered non-profit entity, a News Entity, or an Internet Publishing Organization that:

  - **Prior (Eligible) Borrower** – (i) has received a PPP loan and (ii) on or before the expected date on which a Second Draw Loan is disbursed, has used, or will use full amount of such PPP loan (the Second Draw Rules clarified that (A) "the full amount" of the borrower's PPP loan includes the amount of any increase on such PPP loan made pursuant to the Economic Aid Act and (B) the borrower must have spent the full amount of its PPP loan on eligible expenses under the PPP rules to be eligible for a Second Draw PPP Loan);

  - **Size Test** – employs not more than **300 employees**; and

  - **Gross Receipts Revenue Test** – demonstrates a loss of **not less than 25%** of gross receipts during at least one quarter in 2020 as compared to the corresponding quarter in 2019. In the case of a borrower of a Second Draw Loan of not more than **$150,000**, this test can be satisfied by submission of a certification that the entity meets the applicable revenue loss requirement and later documentation supporting that the revenue loss standard was in fact met prior to submitting for forgiveness of the borrower's Second Draw Loan. Under the Second Draw Rules, a borrower that was in operation in all four quarters of 2019 is deemed to have experienced the required revenue reduction if it experienced a reduction in annual receipts of 25% or greater in 2020 compared to 2019 and the borrower submits copies of its annual tax forms substantiating the revenue decline. Note that any forgiveness amount of a PPP loan that a borrower received in calendar year 2020 is excluded from a borrower's gross receipts, consistent with the purpose of the Second

Draw Program, which is to deliver additional aid to small businesses that previously received a PPP loan.

- **How to Determine a Borrower's Gross Receipts** – the SBA and Treasury posted new guidance, effective as of January 19, 2021, explaining what "gross receipts" means for for-profit businesses and for non-profit organizations.

    - *For for-profit businesses* – Generally, gross receipts are all revenue in whatever form received or accrued from whatever source (including from the sales of products or services, interest, dividends, rents, royalties, fees, or commissions), reduced by returns and allowances but excluding net capital gains and losses.

      Gross receipts do not include:

        - taxes collected for and remitted to a taxing authority if included in gross or total income, such as sales or other taxes collected from customers;

        - proceeds from transactions between a concern and its domestic or foreign affiliates; and

        - amounts collected for another by a travel agent, real estate agent, advertising agent, conference management service provider, freight forwarder, or customs broker.

      Subcontractor costs, reimbursements for purchases a contractor makes at customer's request, investment income, and employee-based costs such as payroll taxes, may not be excluded from gross receipts.

    - *For non-profit organizations* – Gross receipts means gross receipts within the meaning of section 6033 of the Internal Revenue Code of 1986, which is the gross amount received by the organization during its annual accounting period from all sources without reduction for any costs or expenses including costs of goods or assets sold, cost of operations, or expenses of earning, raising, or collecting such amount. Thus, gross receipts for eligible non-profit organizations includes:

        - The gross amount received as contributions, gifts, grants, and similar amounts without reduction for the expenses of raising and collecting such amount,

        - The gross amount received as dues or assessments from members or affiliated organizations without reduction for expenses attributable to the receipt of such amounts,

        - Gross sales or receipts from business activities (including business activities unrelated to the purpose for which the organization qualifies for exemption, the net income or loss from which may be required to be reported on Form 990-T),

        - The gross amount received from the sale of assets without reduction for cost or other basis and expenses of sale, and

        - The gross amount received as investment income, such as interest, dividend, rents, and royalties.

    - The SBA's affiliation rules and waivers of such rules under the PPP are applicable to the Second Draw Program (except under the Second Draw Program the threshold for eligibility is 300 employees rather than 500 employees). **In calculating its applicable gross receipts, the borrower must include the gross receipts of its affiliates** (unless a waiver of affiliation applies) by adding the gross receipts of the borrower with the gross receipts of each of its affiliates. If the borrower has acquired an affiliate or been acquired as an affiliate during 2020, gross receipts includes the receipts of the acquired or acquiring business. This aggregation applies for the entire period of measurement, not just the period after the affiliation arose. However, if a company acquired a segregable division of another company during 2020, gross receipts do not include the receipts of the acquired division prior to the acquisition. Similarly, the gross receipts of a former affiliate are not included. This exclusion of gross receipts of such former affiliate applies during the entire period of measurement, rather than only for the period after which affiliation ceased. However, if the borrower sold a segregable division during 2020, the gross receipts will continue to include the receipts of the division that was sold.

- **Number of Second Draw Loans:** An eligible borrower may only receive one (1) Second Draw Loan.

- **Ineligible Businesses and Organizations:** The following businesses or organizations are expressly ineligible to receive Second Draw Loans:

    - as with the PPP, businesses that are ineligible to receive SBA loans under 13 C.F.R. 120.110 (other than non-profits and certain religious organizations, which are permitted);

    - any business engaged in political or lobbying activities (including one organized for research or for engaging in advocacy in areas such as public policy or political strategy, or one that describes itself as a think tank in any public documents);

    - any business or entity that is **20%** or more (including as equity shares or a capital or profit interest in an LLC or partnership), directly or indirectly, owned by an entity created in or organized under the laws of or that has significant operations in the People's Republic of China ("PRC") or the Special Administrative Region of Hong Kong ("HK");

    - an entity that retains as a member of the board of directors a person who is a resident of PRC;

    - any person required to submit a registration statement under section 2 of the Foreign Agents Registration Act of 1938; or

    - a person or entity that receives a Shuttered Venue Operators grant under section 24 of the Economic Aid Act.

- **Second Draw Loan Amount:** Generally, an eligible Second Draw Loan borrower can receive a loan amount equal to the product of (i) average total monthly payroll costs incurred or paid during, at the borrower's election, calendar year 2019 or 2020 and (ii) 2.5, subject to a cap of $2.0 million. However, there are a few notable variations on the Second Draw Loan amount calculations:

    - **Seasonal Employers** – as with PPP loans, seasonal employers can instead calculate their average total monthly payments for payroll costs using any 12-week period between February 15, 2019 and February 15, 2020;

    - **New Entities** – businesses or organizations that were not in existence during the one (1) year period preceding February 15, 2020 [xxiii] calculate their average total monthly payments for payroll costs by (i) determining the sum of the total monthly payments for payroll costs paid or incurred as of the date on which the entity applies for a Second Draw Loan and (ii) dividing that total by the number of months in which such payroll costs were paid or incurred; and

    - **NAICS 72 Entities** – businesses in the "accommodation and food services" sector (NAICS code beginning with 72) may receive a maximum loan amount equal to the product of (i) average total monthly payroll costs incurred or paid during, at the borrower's election, calendar year 2019 or 2020 and (ii) **3.5**, subject to a cap of $2 million.

    - **Self-Employed Applicants** – (i) self-employed borrowers with no employees may receive a maximum loan amount equal to the product of (A) the average monthly total net profits or gross income (subject to an annualized $100,000 cap) and (B) 2.5 (or 3.5 for borrowers assigned a NAICS code beginning with 72), subject to a cap of $20,833 (or $29,167 for NAICS code 72 borrowers), and (ii) self-employed borrowers with employees may receive a maximum loan amount, equal to the product of (A) the sum of (i) the average monthly total net profits or gross income (subject to an annualized $100,000 cap) and (ii) the average monthly total payment for employee payroll costs incurred by the borrower and (B) 2.5 (or 3.5 for borrowers assigned a NAICS code beginning with 72), subject to a cap of $2 million.

        - For self-employed applicants that have employees, payroll costs for such employees are calculated using:

            - 2019 or 2020 gross wages and tips paid to such employees with a principal place of residence in the US (using 2019 or 2020 IRS Form 941 Taxable Medicare wages & tips from each quarter) *plus* any pre-tax employee contributions for health insurance or other fringe benefits excluded from Taxable Medicare wages & tips (net of any amounts paid to any individual employee in excess of $100,000 annualized cap); and

            - 2019 or 2020 employer group health, life, disability, vision, and dental insurance contributions and retirement contributions listed on the 2019 or 2020 Form 1040 Schedule C or F and state and local taxes assessed on employee compensation.

- **Single Corporate Group Cap:** While the Economic Aid Act does not specifically address whether the general cap of $20 million of total PPP loans received by a single corporate group will also take into account Second Draw Loans (although it implies that they do not), the

Second Draw Rules provide that businesses that are part of a single corporate group shall in no event receive more than $4,000,000 of Second Draw Loans in the aggregate.

- **Second Draw Loan Application and Documentation Requirements:** The Second Draw Rules provide that the documentation required to substantiate an applicant's payroll cost calculation is generally the same as the documents required for PPP loans. However, no additional documentation to substantiate payroll costs will be required if the applicant (i) used calendar year 2019 figures to determine its initial PPP loan amount, (ii) used calendar year 2019 figures to determine its Second Draw Loan amount, and (iii) the lender for the applicant's Second Draw Loan amount is the same as the lender that made the initial PPP loan. The lender may, however, request additional documentation if, on further review, the lender concludes that it would be useful in conducting the lender's good-faith review of the borrower's loan amount calculation.

- **Necessity:** As with First Draw PPP loans, applicants for Second Draw PPP loans are required to certify that the "*current economic uncertainty makes this loan request necessary to support the ongoing operations of the Applicant*." However, (as Question 46 of the FAQs indicates) because Second Draw PPP loan borrowers must demonstrate that they have had a 25% reduction in gross revenues, all Second Draw PPP loan borrowers will be deemed to have made the required certification concerning the necessity of the loan in good faith.

- **Second Draw Loan to Borrower with Unresolved PPP Loans:** If a borrower's PPP loan is under review by the SBA and/or information in SBA's possession indicates that the borrower may have been ineligible for the PPP loan it received or for the loan amount it received, the lender will receive notification from the SBA when the lender submits an application for a guaranty of a Second Draw Loan and will not receive an SBA loan number until the issue related to the borrower's unresolved PPP loan is resolved.

- **Forgiveness of Second Draw Loans:** The forgiveness rules governing PPP loans (as amended by the Economic Aid Act) are also applicable to Second Draw Loans, including the requirement that 60.0% of the Second Draw Loan amount must be utilized for forgiveness-eligible payroll costs. Under the Interim Final Rule published on January 19, 2021, for Second Draw Loans in excess of $150,000, the borrower must submit its loan forgiveness application for the First Draw PPP loan before or simultaneously with the loan forgiveness application for the Second Draw Loan, even if the calculated amount of forgiveness on the First Draw PPP loan is zero. Note that a Second Draw Loan borrower with a principal amount of $150,000 or less is required to provide documentation of revenue reduction if such documentation was not provided at the time of loan application.

- **Lender Fee Limits:** Processing fees paid to lenders will be based on the balance of the loan outstanding at the time of final disbursement. Second Draw Loans are subject to the following adjusted fee scale for lender processing reimbursement: (i) the lesser of $2,500 and 50 percent of the balance of the financing outstanding at the time of disbursement, for loans of not more than $50,000, (ii) 5.00% for loans of more than $50,000 and not more than $350,000; and (iii) 3.00% for loans of more than $350,000.

  The Review Rules provide that if the SBA conducts a review of a PPP loan and determines that a borrower is ineligible, then the lender is not eligible for a processing fee. Lender fees are subject to claw-back within 1 year of disbursement of a PPP loan if the SBA determines that a borrower was ineligible. However, if the lender has received a processing fee on a loan

that was cancelled or voluntarily terminated and repaid after disbursement (including if a borrower repaid the PPP loan proceeds to conform to the borrower's certification regarding the necessity of the PPP loan request), the SBA will not require the lender to repay the processing fee unless the lender is found guilty of an act of fraud in connection with the PPP loan.

III. Tax Matters

The following summarizes key tax provisions of the CAA relevant to the PPP and related matters, including the ERTC. For additional information on the tax provisions contained in the CAA more broadly please see our *Tax Talks* blog post *Coronavirus: President Trump Signs Consolidated Appropriations Act, 2021; Summary of the Tax Provisions*.

- **Expenses Related to PPP Loan Forgiveness are Deductible:** Under the CARES Act, the forgiveness of a PPP loan does not give rise to taxable cancellation of indebted income, or a loss of tax attributes. However, the IRS had held that expenses that gave rise to PPP loan forgiveness were *not* deductible. The CAA reverses this rule and permits taxpayers whose PPP loans are forgiven to deduct the expenses relating to their loans to the extent they would otherwise qualify as ordinary and necessary business expenses. This rule applies retroactively to the effective date of the CARES Act so that expenses paid using funds from PPP loans previously issued under the CARES Act are deductible regardless of when the loan was forgiven.

- **Eligibility for the Employee Retention Tax Credit:** The CARES Act provided an eligible employer with a refundable payroll tax credit equal to 50% of certain "qualified wages" (including certain health plan expenses) paid to its employees beginning March 13, 2020 through December 31, 2020 if the employer is engaged in a trade or business in 2020 and the wages are paid (i) while operation of that trade or business is fully or partially suspended due to a governmental order related to COVID-19 (the "suspension test") or (ii) during the period beginning in the first quarter in which gross receipts for that trade or business are less than 50% of gross receipts for the same calendar quarter of 2019 and ending at the end of the first subsequent quarter in which gross receipts are more than 80% for the same calendar quarter of 2019 (the "gross receipts test"). The ERTC can be used to offset all federal payroll taxes, including federal withholding taxes, and the employer's and employee's share of social security tax and Medicare, but not the federal unemployment tax.

  The CAA makes several changes to the ERTC and extends the availability of the ERTC through July 1, 2021. As discussed briefly above, one such change is that, whereas the CARES Act denied the ERTC to any employer that receives a PPP loan, and defined the term "employer" expansively, potentially causing acquiring corporations with ERTCs to lose or recapture those tax credits if they acquired a target company that had received a PPP loan, the CAA instead permits an employer that receives a PPP loan to receive the ERTC. However, to prevent any double-dipping, an employer must either exclude "qualified wages" that allowed the employer to claim ERTCs from "payroll costs" for purposes of determining its loan forgiveness under the PPP (so as to reduce the amount of loan forgiveness), or exclude "payroll costs" that qualified for PPP loan forgiveness from "qualified wages" (so as to reduce the ERTC). This change applies retroactively to the effective date of the CARES Act.

  Additional changes (each of which is effective for calendar quarters beginning after December 31, 2020) are highlighted in the following chart and summarized below.

	CARES Act	CAA
Limitations on credit	50% of qualified wages; $5,000 annual cap	70% of qualified wages; $28,000 annual cap
Eligibility of PPP borrowers	Not eligible	Eligible (with election to exclude qualified wages from either payroll costs for PPP purposes or from calculation of credit). As noted above, this applies retroactively to the effective date of the CARES Act.
Large employer threshold for additional limitations	100 full-time employees	500 full-time employees
Governmental organizations	Not eligible	Certain governmental instrumentalities are eligible
Availability	Through December 31, 2020	Through July 1, 2021

- **Credit increased to 70% of qualified wages; cap on credit increased to $28,000** – The CARES Act provided for a refundable payroll tax credit of 50% of certain "qualified wages," capped at $5,000/employee (50% of up to $10,000 of qualified wages for all calendar quarters). The CAA increases the credit cap from $5,000 for the year to $7,000 (70% of $10,000) for any calendar quarter. Accordingly, the CAA will increase the maximum amount of credit available in 2021 for each employee from $5,000 to $28,000.

- **Helpful changes to the gross receipts test** – Under the CARES Act, an employer qualified for the ERTC under the gross receipts test for the period beginning with the first calendar quarter for which gross receipts for the employer's trade or business were less than 50% of gross receipts for the same calendar quarter of 2019 and ending at the end of the first subsequent quarter in which gross receipts were more than 80% for the same calendar quarter of 2019. Under the CAA, in 2021, an employer now qualifies for the period beginning in a calendar quarter in which the employer's gross receipts are less than 80% (instead of 50%) of gross receipts for the same calendar quarter of 2019 and ending at the end of the first subsequent quarter in which gross receipts are more than 80% for the same calendar quarter of 2019. Furthermore, employers may elect to apply the gross receipts test based on gross receipts from the prior calendar quarter to determine their eligibility for the ERTC. The CAA also makes the ERTC available in 2021 to employers that were not in existence in 2019 by permitting them to apply the gross receipts test based on 2020 gross receipts.

- **Large employer threshold** – For an employer with more than 100 full-time employees, the CARES Act imposed an additional restriction: the ERTC is available only with respect to wages paid to an employee who is not providing services due to circumstances described in (i) the suspension test or (ii) the gross receipts test. The CAA increases the threshold for this rule in 2021 to 500 full-time employees (so that employers with between 101 and 500 full-time employees would no longer be subject to this restriction).

- **Tax-exempt organizations and governmental entities** –

    - The CAA provides that, for purposes of the ERTC, the term "gross receipts" of a section 501(c) tax-exempt organization means the amounts the organization

receives during its annual accounting period from all sources without subtracting any costs or expenses. This change applies retroactively to the effective date of the CARES Act.

- Under the CARES Act, federal, state, or local governments (and their agencies) were not eligible for the ERTC. The CAA permits federal credit unions, public colleges and universities, and public medical and healthcare providers to receive the ERTC if they otherwise satisfy the requirements for the credit. For purposes of determining eligibility, public colleges and universities and public medical and healthcare providers are treated as being engaged in a trade or business.

- **Health plan expenses** – The CAA includes health plan expenses in the definition of "qualified wages" for purposes of the ERTC, including in cases where an employer furloughs employees but continues to provide health benefits to them. This change applies retroactively to the effective date of the CARES Act.

- **Social Security Tax Deferral:** The CARES Act permits employers to delay payment of the 6.2% **employer share** of the Social Security tax (but not the 1.45% employer share of the Medicare tax) from the date of enactment through December 31, 2020. The tax is payable over the following 2 years with half paid by December 31, 2021 and the other half by December 31, 2022. However, the deferral is **not** available for an employer who has a PPP loan forgiven. The deferral of the employer's share of the Social Security tax was not extended by the CAA. On August 8, 2020, President Trump issued a memorandum permitting employers to defer payment of the Social Security portion of payroll taxes (i.e., the Old-Age, Survivors, and Disability Insurance tax under Section 3101(a) and Railroad Retirement Act Tier 1 tax under Section 3201(a)) for any employee with pre-tax wages or compensation during any biweekly pay period that were less than $4,000 during the period between September 1, 2020 and December 31, 2020. The memorandum required the employers to withhold and pay the deferred payroll taxes from wages or compensation paid between January 1, 2021 and April 30, 2021. The CAA extends the repayment period to December 31, 2021. The longer period will result in less of the deferred amounts being subtracted from each paycheck.

IV. Frequently Asked Questions

- **Q1: What affiliation rules apply (for purposes of determining the number of employees of an applicant together with its affiliates)?** [xxiv]

A: According to the U.S. Treasury Department's affiliation guidance, the four affiliation tests below are applicable to an affiliation assessment for purposes of determining eligibility under the PPP. The Treasury Department's guidance (combined with language in the CARES Act rescinding the SBA's February 2020 Interim Final Rule on affiliation standards) confirms that the pre-2020 SBA rules on affiliation (13 C.F.R. § 121.301(f)(1) – (4)) are the relevant affiliation rules for purposes of the PPP:

- Affiliation based on ownership;
- Affiliation arising under stock options, convertible securities, and agreements to merge;
- Affiliation based on management; and

- Affiliation based on identity of interest between "close relatives."

- **Q2: When is a minority shareholder deemed to have control (and therefore affiliation)?**

  A: The SBA distinguishes between rights in respect of ordinary business actions and "extraordinary" business actions necessary to protect the minority investor's investment. In instances where supermajority consent is required for **ordinary business actions**, the minority investor's ability to block such actions gives rise to negative control and the investor will be **deemed an affiliate**. In contrast, a minority investor's ability to block **"extraordinary" business actions** should **not** give rise to affiliation between a minority investor and the applicant. **Please note that this distinction is derived from SBA case law, not all of which is specific to the affiliation rules for 7(a) loan programs (like the PPP). Applicants are strongly encouraged to carefully assess any minority protections before determining that such protections do not give rise to affiliation**.

Examples of minority rights that have been determined to establish **control** by the minority investor and **result in affiliation** include the following:	Examples of minority rights that are with respect to "extraordinary" business actions and have been determined **not to establish control** (and thus, **no affiliation**) include the following:
• Making, declaring, or paying distributions or dividends other than tax distributions; • Establishing a quorum at a meeting of stockholders (and likely, by extension, at a meeting of the board); • Approving or making changes to the company's budget or approving capital expenditures outside the budget; • Determining employee compensation; • Hiring and firing officers and executives; • Blocking changes in the company's strategic direction; • Establishing or amending an incentive or employee stock ownership plan; • Incurring or guaranteeing debts or obligations; • Initiating or defending a lawsuit; • Entering into contracts or joint ventures; and • Amending or terminating leases.	• Selling all or substantially all of the company's assets; • Placing an encumbrance or lien on all or substantially all of the company's assets; • Engaging in any action that could result in a change in the amount or character of a company's capital contributions; • Changing the company's line of business; • Engaging in a merger transaction (only applies to veteran-owned businesses); • Issuing additional stock/equity; • Amending the organizational documents of a company; • Filing for bankruptcy; • Amending the governing documents to materially alter the rights of the existing owners; • Dissolving the company; • Increasing, decreasing, or reclassifying the authorized capital of the company;

Examples of minority rights that have been determined to establish **control** by the minority investor and **result in affiliation** include the following:	Examples of minority rights that are with respect to "extraordinary" business actions and have been determined **not to establish control** (and thus, **no affiliation**) include the following:
	• Taking an action in contravention of a company's charter, bylaws, operating agreement, or similar governing documents; • Disposing of the company's goodwill; • Committing any act that would make it impossible for the company to carry on its ordinary course of business; • Submitting a company's claim to arbitration; • Entering into a confession of a judgment; • Adding new members; and • Approving an increase or decrease in the size of the company's board of directors or other governing body.

The SBA has confirmed that a minority shareholder can eliminate such affiliation if such shareholder "irrevocably waives or relinquishes" such rights.

- **Q3: When does a management agreement create "control"?**

  A: Management agreements that give the management company sole discretion over the business operations with minimal oversight of the decision-making by the applicant, while not passive, create affiliation between the management company and applicant. However, affiliation is not created between the applicant and the management company if the management agreement includes "meaningful oversight" by the applicant over the management company's activities. A management agreement that provides for the applicant business to do all of the following inherently provides for "meaningful oversight": (i) approval of the annual operating budget; (ii) approval of any capital expenditures or operating expenses over a significant dollar threshold; (iii) control over bank accounts; and (iv) oversight over the employees operating the business.

- **Q4: In addition to the Form 2483, what other documentation are lenders asking for?**

  - ***A:*** Lenders have generally requested the following, though they may request additional or alternative materials:
    - IRS 940, 941, or 944 payroll tax forms for 2019, and if available, Q1 2020;

- Payroll processor records and other payroll reports/ledgers for 2019 and 2020 with corresponding bank statements (which should capture the following information: salary, wages, commission, or similar compensation; tips; vacation; parental, family, medical, or sick leave; group healthcare benefits; retirement benefits; and state or local taxes on employee compensation);
- 1099s for independent contractors;
- Documentation evidencing health insurance premiums under a group health plan;
- Documentation evidencing the sum of all retirement plan funding paid for by the applicant; and
- Organizational documents (articles of incorporation/organization, bylaws, operating agreement, partnership agreement, owners' driver's licenses, etc.) and tax identification numbers (EINs, SSNs or ITINs, as appropriate).

- **Q5: What non-profits are eligible for the PPP?**

  A: Under the Cares Act, the Economic Aid Act, and the PPP Rules, the tax-exempt non-profit organizations described in section 501(c)(3) of the Internal Revenue Code (the "IRC"), the tax-exempt veterans organizations described in section 501(c)(19) of the IRC, News Entities, Eligible 501(c)(6) Organizations, and Destination Marketing Organizations are eligible for the PPP.

- **Q6: What information about PPP borrowers will/have become publicly available?**

  A: Requests for information about a borrower may be denied unless the SBA has the written permission of the borrower or the information is subject to disclosure under the Freedom of Information Act (FOIA). FOIA requires the SBA to disclose, upon request, information supplied by borrowers as part of loan programs upon request, including:

  - Statistics on the PPP (individual borrowers are not identified in the statistics) and
  - Borrower information including: (i) names and commercial street and e-mail addresses; (ii) names of officers, directors, stockholders, or partners; and (iii) loan amount.

  Treasury and the SBA have published loan-level data for PPP loans, which data is available for download on Treasury's website. This data is bifurcated into PPP loans with a principal amount that is $150,000 and greater and those with a principal amount that is less than $150,000 and further categorized by state.

  For PPP loans with a principal amount that is $150,000 and greater, the available data includes for each loan: the loan amount range; borrower's name, address, NAICS code, and legal entity type; demographic data (as to race/ethnicity, gender, and veteran status, which was optional to provide in the loan application); number of jobs retained; loan approval date; and the lender name. Nationwide data for such loans is aggregated in a single spreadsheet.

  For PPP loans with a principal amount that is less than $150,000, the name of the borrower is not disclosed. The available data includes for each loan: the specific loan amount;

borrower's city, state, zip code, NAICS code, and legal entity type; (optional) demographic data; number of jobs retained; loan approval date; and the lender name. Data for such loans can be downloaded for each state.

Proprietary data on a borrower is not routinely made available to third parties, and commercial or financial information obtained from a person is exempt from FOIA requests. Further, according to the SBA, materials and information *generally* exempt from FOIA requests include: financial statements; credit reports; business plans; fiscal projections; pricing or payroll information; corporate structures; personal and business tax returns; non-statistical information on pending, declined, withdrawn, or cancelled applications or on defaults or delinquencies; requests for size determinations; loan applications; and loan officers' reports (among other materials and information). Under the Privacy Act, the SBA is also authorized to make certain "routine uses" of information protected by that Act (*e.g.*, disclosure of information maintained in SBA's records when it indicates a violation or potential violation of law to the appropriate Federal, State, local, or foreign enforcement agency).

- **Q7: What should a borrower do if a rule change (or FAQ) alters a borrower's eligibility?**

  A: While there is greater clarity now around the risks associated with the necessity certification, there remains a broader issue of what actions an existing borrower must take when a PPP Rule or FAQ that alters or clarifies PPP eligibility would result in that borrower being ineligible. Question 17 of the SBA FAQs provides that borrowers "may rely on the laws, rules, and guidance available at the time of the relevant application" and do not need to take action based on updated guidance. However, leaning on Question 17 comes with potentially serious pitfalls. First, FAQ is not law or part of an Interim Final Rule, so it is uncertain how much weight an FAQ carries. Second, it is unclear whether the SBA draws a meaningful distinction between a *new* law, rule, or guidance that is a true change in the PPP as compared to a clarification, or less, a reassertion of an existing rule. The government may also take the position that the May 18, 2020 safe harbor period, while purportedly applying only to the necessity certification, allowed borrowers the opportunity to return funds and any borrower who chose not to do so, in effect, recertified that it was eligible for a PPP loan. A borrower whose eligibility is in question that retained its PPP loan after May 18, 2020 may ultimately have to repay loan proceeds in full (potentially immediately or on an expedited basis) and perhaps even incur criminal and civil penalties (*e.g.,* if a borrower has to re-certify as to eligibility in a forgiveness application) (see "**Consequences of a False Filing**" above).

- **Q8: In addition to the Form 3508 (as revised on June 16, 2020) (or Forms 3508EZ or 3508S), what other materials must be submitted as part of the loan forgiveness application and for how long must such materials be retained?**

  A: Loans that are not more than $150,000 may be forgiven if the PPP borrower submits a one page certification, Form 3508S, which, consistent with the requirements of the Economic Aid Act, contains: (i) the number of employees the eligible recipient was able to retain because of the covered loan, (ii) the estimated amount spent on payroll costs, and (iii) the total loan value. By completing this form, the PPP borrower attests that the information provided is accurate and that it complied with the requirements under section 7(a)(36). Such PPP borrower is required to retain records that affirm compliance with such requirements (as to employment records, for the four year period following submission, and for other records, for the three year period following submission). The

Economic Aid Act expressly states that such PPP borrower is not required to submit as part of the forgiveness application process any additional application or documentation to substantiate forgiveness. This new easy application process for loans of not more than $150,000 has retroactive effect, and applies to existing PPP loans and PPP loans made on or after the enactment of the Economic Aid Act. The SBA can review and audit such loans and access any records the borrower is required to retain.

The Economic Aid Act provides that for loans in excess of $150,000, borrowers must generally submit the following (which expands upon the application requirements described in the CARES Act):

- documentation verifying the number of full-time equivalent employees on payroll and pay rates for the periods described in subsection (d) thereof, including —
    - (A) payroll tax filings reported to the Internal Revenue Service; and
    - (B) State income, payroll, and unemployment insurance filings;
- documentation, including cancelled checks, payment receipts, transcripts of accounts, purchase orders, orders, invoices, or other documents verifying payments on covered mortgage obligations, payments on covered rent obligations, payments on covered operations expenditures, payments on covered property damage costs, payments on covered supplier costs, payments on covered worker protection expenditures, covered lease obligations, and covered utility payments;
- a certification from a representative of the eligible recipient authorized to make such certifications that—
    - (A) the documentation presented is true and correct; and
    - (B) the amount for which forgiveness is requested was used to retain employees, make interest payments on a covered mortgage obligation, make payments on a covered rent obligation, make payments on covered operations expenditures, make payments on covered property damage costs, make payments on covered supplier costs, make payments on covered worker protection expenditures, or make covered utility payments; and
- any other documentation the SBA Administrator determines necessary.

The forgiveness application requirements originally set forth in the CARES Act (and prior to the amendments under the Economic Aid Act) were detailed and clarified in SBA Form 3508 and the instructions to that form.

Payroll Cost Documentation – Documentation verifying the eligible cash compensation and non-cash benefit payments from the covered period, consisting of:

- Bank account statements or third-party payroll service provider reports documenting the amount of cash compensation paid to employees;

- Tax forms (or equivalent third-party payroll service provider reports) for the periods that overlap with the covered (payroll tax filings (*i.e.*, Form 941) and state quarterly business and individual employee wage reporting and unemployment insurance tax filings); and

- Payment receipts, cancelled checks, or account statements documenting the amount of any employer contributions to employee group health, life, disability, vision, or dental insurance and retirement plans that the borrower included in the forgiveness amount.

Non-payroll Costs – Documentation verifying existence of the obligations/services prior to February 15, 2020 and eligible payments from the covered period:

- **Business Mortgage Interest Payments** – Copy of lender amortization schedule and receipts or cancelled checks verifying eligible payments from the covered period; or lender account statements from February 2020 and the months of the covered period through one month after the end of the covered period verifying interest amounts and eligible payments;

- **Business Rent or Lease Payments** – Copy of current lease agreement and receipts or cancelled checks verifying eligible payments from the covered period or lessor account statements from February 2020 and from the covered period through one month after the end of the covered period verifying eligible payments;

- **Business Utility Payment –** Copy of invoices from February 2020 and those paid during the covered period and receipts, cancelled checks, or account statements verifying those eligible payments;

- **Covered Operations Expenditures** – Copy of invoices, orders, or purchase orders paid during the covered period and receipts, cancelled checks, or account statements verifying those eligible payments;

- **Covered Property Damage Costs** – Copy of invoices, orders, or purchase orders paid during the covered period and receipts, cancelled checks, or account statements verifying those eligible payments, and documentation that the costs were related to property damage and vandalism or looting due to public disturbances that occurred during 2020 and such costs were not covered by insurance or other compensation;

- **Covered Supplier Costs** – Copy of contracts, orders, or purchase orders in effect at any time before the covered period (except for perishable goods), copy of invoices, orders, or purchase orders paid during the covered period, and receipts, cancelled checks, or account statements verifying those eligible payments;

- **Covered Worker Protection Expenditures** – Copy of invoices, orders, or purchase orders paid during the covered period and receipts, cancelled checks, or account statements verifying those eligible payments, and documentation that the expenditures were used by the borrower to comply with applicable COVID-19 guidance during the covered period.

- **FTE Reference Period Documentation –** Documentation showing the average number of FTE employees on payroll per week employed by the borrower during the selected reference period (see "**Reductions in Forgiveness Amount**" **above**). Such documentation may include payroll tax filings and state quarterly business and individual employee wage reporting and unemployment insurance tax filings.

- **Borrower is <u>not</u> required to submit (but must retain) the PPP Schedule A Worksheet included in Form 3508** (which is used to calculate average FTE during the covered period, list salary and compensation paid to employees during the covered period, confirm whether any related reductions to the forgiveness amount are required, and confirm whether any such reductions fall within the safe harbor exceptions) **and related documentation supporting the calculations in such worksheet, including (as clarified in Form 3508), if applicable:**

    - regarding any employee job offers and refusals, refusals to accept restoration of reductions in hours, firings for cause, voluntary resignations, written requests by any employee for reductions in work schedule, and any inability to hire similarly qualified employees for unfilled positions on or before (i) December 31, 2020 for a PPP loan made before December 27, 2020 or (ii) the last day of the covered period for a PPP loan made on or after December 27, 2020; and

    - supporting the certification that the borrower was unable to operate between February 15, 2020 and the end of the covered period at the same level of business activity as before February 15, 2020, due to compliance with requirements established or guidance issued between March 1, 2020 and December 31, 2020 (or for a PPP loan made on or after December 27, 2020 requirements established or guidance issued between March 1, 2020 and the last day of the covered period) by the Secretary of Health and Human Services, the Director of the Centers for Disease Control and Prevention, or the Occupational Safety and Health Administration, related to the maintenance of standards of sanitation, social distancing, or any other work or customer safety requirement related to COVID-19. This documentation must include copies of the applicable requirements for each borrower location and relevant borrower financial records.

    - The borrower must retain all such documentation in its files for 6 years after the date the loan is forgiven or repaid in full, and permit authorized representatives of SBA, including representatives of its Office of Inspector General, to access such files upon request.

- **Demographic Information** – Borrowers can complete an optional form on certain demographic information (including gender, race, ethnicity, and veteran status/relationship).

- **PPP Loan Necessity Questionnaires** – In late 2020, the SBA produced loan necessity questionnaires for each of for-profit borrowers (SBA Form 3509) and non-profit borrowers (SBA Form 3510) that must be completed and submitted by each PPP borrower that together with its affiliates received PPP loans with an original principal amount of $2 million or greater to such PPP borrower's lender within ten business days of receipt of such from such lender.

- **SBA Form 3508D** – If Form 3508 or Form 3508EZ is being submitted for a First Draw PPP loan approved on or before August 8, 2020, and the borrower is required to submit SBA Form 3508D, Form 3508D must also be submitted to the SBA as part of forgiveness application. For loans made prior to December 27, 2020, if the President of the U.S., Vice President of the U.S., the head of an Executive department, or a Member of Congress, or the spouse of such person as determined under applicable common law, directly or indirectly holds a controlling interest in a borrower, the borrower must disclose this information to the SBA by submitting Form 3508D. Such disclosure must be made no later than January 26, 2021, if the borrower submitted an application for forgiveness before December 27, 2020, or no later than 30 days after submitting an application for forgiveness.

- **Q9: Which borrowers are eligible to utilize SBA Form 3508EZ?**

  ***A*:** SBA Form 3508EZ is a simplified forgiveness application that can be utilized by a borrower that certifies that it falls into one of the two below categories. Form 3508EZ generally tracks Form 3508 and has similar documentation requirements, but eliminates steps that address the reduction in the forgiveness amount due to reductions in employee compensation or in FTE levels.

  - **Category 1:** Borrower did not reduce:

    - (1) annual salary or hourly wages of any employee (that did not receive, during any single period during 2019, wages or salary at an annualized rate of pay in an amount more than $100,000) by more than 25% during the covered period compared to the most recent full quarter before the covered period ; **AND**

    - (2) the number of employees or the average paid hours of employees between January 1, 2020 and the end of the covered period. Form 3508EZ expressly states that in assessing eligibility under this second prong, borrowers should ignore reductions that arose from (i) an inability to rehire individuals who were employees on February 15, 2020 if the borrower was unable to hire similarly qualified employees for unfilled positions on or before December 31, 2020 (or for a PPP loan made on or after December 27, 2020, the last day of the covered period), and (ii) hours that the borrower offered to restore and the employee refused.

  - **Category 2:**

    - (1) Borrower did not reduce annual salary or hourly wages of any employee (that did not receive, during any single period during 2019, wages or salary at an annualized rate of pay in an amount more than $100,000) by more than 25% during the covered period compared to the last full quarter before the covered period; **AND**

    - (2) Borrower was unable to operate during the covered period at the same level of business activity as before February 15, 2020, due to compliance with requirements established or guidance issued between March 1, 2020 and December 31, 2020 (or for a PPP loan made on or after December 27, 2020, requirements established or guidance issued between March 1, 2020

and the last day of the covered period) by the Secretary of Health and Human Services, the Director of the Centers for Disease Control and Prevention, or the Occupational Safety and Health Administration, related to the maintenance of standards of sanitation, social distancing, or any other work or customer safety requirement related to COVID-19.

- **Q10: Should lenders report forgiveness of PPP loans to the IRS on Form 1099-C?**

  ***A*:** Many lenders that forgive more than $600 of a borrower's debt are ordinarily required to file a notice of the forgiveness with the IRS on "Form 1099-C, Cancellation of Debt" and issue a related statement to the borrower. Among other things, the filings allows the IRS to track and confirm that cancellation of indebtedness income is being properly reported by borrowers. Where a PPP borrower satisfies the statutory PPP loan program forgiveness requirements and obtains forgiveness, however, the forgiven loan amount is excluded from the PPP borrower's gross income. Announcement 2020-12, the IRS clarified that the Form 1099-C and payee statement filings for regular loan forgiveness are not required where a PPP loan is forgiven because the borrower satisfies the statutory forgiveness requirements, and, indeed, that a lender that so forgives a PPP loan "should not" make these filings for the loan. The announcement states that this approach is intended to prevent confusion.

- **Q11: What happens if a borrower or lender committed an error that caused a borrower to receive a PPP loan amount that exceeds the borrower's correct maximum loan amount?**

  ***A*:** According to SBA's recent guidance, if the error was caused through a knowing misstatement in the loan application, the borrower may face a fraud charge. However, if a borrower in good faith mistakenly and incorrectly filled out the PPP loan application form and this resulted in the borrower receiving a PPP loan amount that exceeds the borrower's correct maximum loan amount, the borrower will not receive loan forgiveness for any amount that exceeds the correct maximum loan.

- **Q12: When is a borrower no longer considered to be presently involved in a bankruptcy proceeding if the borrower has filed a bankruptcy petition in the past?**

  ***A*:** According to Question 67 of SBA's FAQs: (i) if a borrower has filed a Chapter 7 bankruptcy petition, the borrower is considered to be presently in a bankruptcy proceeding until the Bankruptcy Court has entered a discharge order in the case; (ii) if a borrower has filed a Chapter 11, 12 or 13 bankruptcy petition, the borrower is considered to be presently in a bankruptcy proceeding until the Bankruptcy Court has entered an order confirming the plan in the case; and (iii) regardless of Chapter, if the Bankruptcy Court has entered an order dismissing the case, the borrower is no longer presently in a bankruptcy proceeding. The discharge order, order confirming the plan or order dismissing the case, as applicable, must be entered prior to the date of the PPP loan application. Notwithstanding the foregoing, a borrower is ineligible for a PPP loan if, as a result of a bankruptcy filing, the borrower has permanently closed.

V. Overview of the Paycheck Protection Program Liquidity Facility

On April 9, 2020 the Board of Governors of the Federal Reserve System introduced the Paycheck Protection Program Liquidity Facility (the "PPPLF") pursuant to section 13(3) of the Federal Reserve Act. The PPPLF came as part of a broader announcement by the Federal Reserve and Treasury regarding the implementation of new and expansion of existing Federal lending programs, including the now-terminated Main Street Lending Program aimed at making new loans available to small and medium sized businesses. Notably, the guidance provided that a borrower under the PPP could also borrow under the Main Street Lending Program. (For more on the Main Street Lending Program, see our client alert: *Where is Main Street?—Fed Provides Guidance on the Main Street Lending Program*.)

The terms of the PPPLF are summarized in a term sheet released by the Federal Reserve in conjunction with its announcement, and further detailed in frequently asked questions published by the Federal Reserve. The purpose of the PPPLF is to increase liquidity for lenders participating in the PPP (a "PPP Lender")[xxv] so that they can engage in more expansive origination of PPP loans. Under the PPPLF, Federal Reserve Banks will extend credit to PPP Lenders in the form of non-recourse[xxvi] term loans ("PPPLF Loans") at an interest rate of 0.35%. PPP loans will serve as collateral for a corresponding PPPLF Loan (with such collateral valued at the principal amount of the PPP loan). PPP Lenders can borrow from the PPPLF an amount up to the principal amount of PPP loan collateral that it can pledge to the Federal Reserve. On April 30, 2020, the Federal Reserve confirmed that PPP Lenders will be able to pledge as collateral not only PPP loans that they originate, but also PPP loans acquired on the secondary market.

PPP Lenders seeking PPPLF Loans are required to pool all PPP loans that have the same maturity date, and will receive a single extension of credit secured by such pooled PPP loans. PPP Lenders will need to ensure that they simultaneously pledge all PPP loans with the same maturity date. There will be a separate extension of PPPLF credit for each maturity date of PPP loans that are pledged as collateral. PPP loans cannot be pledged as collateral until the PPP loan has been originated, and cannot be pledged in advance for an extension of credit at a later date.

The terms of a PPPLF Loan will be closely aligned with the underlying PPP loans serving as collateral. The principal amount and maturity period of a PPPLF Loan will be the same as that of the underlying pool of PPP loans. A PPP Lender is required to repay an extension of credit under the PPPLF whenever (i) the PPP Lender has been reimbursed by the SBA for loan forgiveness (to the extent of the forgiveness), (ii) the PPP Lender has received payment from the SBA representing exercise of a loan guarantee, or (iii) the PPP Lender has received payment from the PPP borrower of an underlying PPP loan. In each such instance, the PPP Lender must promptly report to the lending Federal Reserve Bank any payments on pledged PPP loans so that the corresponding PPPLF Loan can be adjusted accordingly. The maturity of a PPPLF Loan will accelerate (i) in conjunction with the acceleration of an underlying PPP loan upon a default and resulting sale to the SBA by the PPP Lender of such PPP loan to realize on the 100% SBA guarantee, and (ii) to the extent of any loan forgiveness reimbursement received by a PPP Lender from the SBA in respect of the underlying PPP loan.[xxvii] PPP Lenders are not required to pay any fees to participate in the PPPLF and there are no prepayment penalties.

A PPP Lender seeking a PPPLF Loan must execute a PPPLF Letter of Agreement and make a certification that (i) it is not insolvent and (ii)[xxviii] it is unable to secure adequate credit accommodations from other banking institutions.[xxix] The Federal Reserve, through its discount window site, has produced: (i) a page with all information and guidance regarding the PPPLF for depository institutions; and (ii) a page with all information and guidance regarding the PPPLF for non-depository institutions.

The Federal Reserve publicly discloses certain information regarding the PPPLF. The Federal Reserve will report weekly (on an aggregate nationwide basis) balance sheet items related to the PPPLF. Further, the Federal Reserve produces a monthly report regarding the CARES Act-related lending facilities, including the PPPLF, detailing (i) names and details of participants in each facility, (ii) amounts borrowed and interest rate charged, and (iii) overall costs, revenues, and fees for each facility. Similar information will also be publicized by the Federal Reserve one year after the termination of the PPPLF. Such reports and information are available on the Federal Reserve's PPPLF page.

All depository institutions that originate PPP loans are eligible to borrow under the PPPLF. On April 30, 2020, the Federal Reserve confirmed that other SBA-qualified PPP lenders, including depository institutions (*i.e.*, banks and credit unions) and non-depository institutions, such as community development financial institutions, small business lending companies licensed by the SBA, and some financial technology firms are eligible to borrow under the PPPLF.

The PPPLF has been extended and will remain in effect until June 30, 2021, unless further extended by Treasury and the Federal Reserve.

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Proskauer's cross-disciplinary, cross-jurisdictional Coronavirus Response Team is focused on supporting and addressing client concerns. We will continue to evaluate the CARES Act, the Consolidated Appropriations Act, 2021, related rules and regulations and any subsequent legislation to provide our clients guidance in real time. Please visit our Coronavirus Resource Center for guidance on risk management measures, practical steps businesses can take and resources to help manage ongoing operations.

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[i] Of the additional appropriated $310 billion under the PPHCEA, $60 billion was expressly allocated for guarantees of loans made by smaller banks, smaller credit unions, and community financial institutions (which encompass certain community development financial institutions, minority depository institutions, and other institutions that provide financing to underserved and economically disadvantaged communities). The PPPHCEA also increased the funding available for the SBA's economic injury disaster loan ("EIDL") program ($50 billion) and for the EIDL grant program introduced in the CARES Act ($10 billion). Additionally, the PPPHCEA appropriated a total of $100 billion to the Public Health and Social Services Emergency Fund, including $75 billion to be distributed by the U.S. Department of Health and Human Services to certain eligible healthcare providers (*e.g.*, hospitals) to reimburse expenses and lost profits attributable to coronavirus. Read more in our client alert on the health care funding under the PPPHCEA and the CARES Act (including grants from the Provider Relief Fund).

[ii] Section 37 of Section 7(a) of the Small Business Act.

[iii] The "PPP Rules" include (a) the Interim Final Rules: (i) an Interim Final Rule governing the PPP generally (published April 2, 2020); (ii) an Interim Final Rule regarding the application of the SBA's affiliation rules to the PPP (published April 2, 2020); (iii) an Interim Final Rule regarding additional eligibility criteria and requirements for certain pledges of loans (with a principal focus on certain self-employed applicants) (published April 14, 2020), (iv) an Interim Final Rule regarding certain requirements for promissory notes, authorizations, affiliation, and eligibility (published April 24, 2020); (v) an Interim Final Rule on additional

criterion for seasonal employers; (vi) an Interim Final Rule on disbursements (published April 28, 2020); (vii) an Interim Final Rule on corporate groups and non-bank and non-insured depository institution lenders (published April 30, 2020); (viii) an Interim Final Rule on nondiscrimination and additional eligibility criteria (published May 5, 2020); (ix) an Interim Final Rule regarding extension of the limited safe harbor with respect to certification concerning need for PPP loan request (published May 8, 2020); (x) an Interim Final Rule on loan increases (published May 13, 2020); (xi) an Interim Final Rule on eligibility of certain electric cooperatives (published May 14, 2020); (xii) an Interim Final Rule on the treatment of entities with foreign employees; (xiii) an Interim Final Rule on the second extension of the limited safe harbor with respect to the necessity certification and lender reporting (published May 20, 2020); (xiv) an Interim Final Rule on Loan Forgiveness (published May 22, 2020), (xv) an Interim Final Rule on SBA loan review procedures and related borrower and lender responsibilities (published May 22, 2020), (xvi) an Interim Final Rule on the eligibility of certain telephone cooperatives (published June 5, 2020), (xvii) an Interim Final Rule amending the first Interim Final Rule in light of the PPPFA (published June 11, 2020), (xviii) an additional Interim Final Rule amending the first Interim Final Rule (published June 12, 2020), (xix) an Interim Final Rule (published June 17, 2020) amending the third (published April 14, 2020) and the sixth (published April 28, 2020) Interim Final Rules, (xx) an Interim Final Rule (published June 22, 2020) amending the fourteenth (published May 22, 2020) and fifteenth (published May 22, 2020) Interim Final Rules, (xxi) an Interim Final Rule (published June 24, 2020) amending the first Interim Final Rule (published April 2, 2020), (xxii) an Interim Final Rule on certain eligible payroll costs (published June 25, 2020), (xxiii) an Interim Final Rule on Appeals of SBA Loan Review Decisions Under the PPP (published August 11, 2020), (xxiv) an Interim Final Rule (published October 8, 2020) providing for additional revisions to loan forgiveness Interim Final Rule (published May 22, 2020 and amended June 22, 2020) and SBA loan review procedures and related borrower and lender responsibilities Interim Final Rule (published May 22, 2020 and amended June 22, 2020), (xxv) an Interim Final Rule (published January 6, 2021) incorporating the Economic Aid Act amendments required to be implemented by regulation within 10 days of enactment of Economic Aid Act, (xxvi) an Interim Final Rule (published January 6, 2021) implementing the key provisions of section 311 of the Economic Aid Act regarding Paycheck Protection Program Second Draw Loans, (xxv) an Interim Final Rule (published January 19, 2021) on loan forgiveness requirements and loan review procedures as amended by the Economic Aid Act, consolidating prior rules related to forgiveness and review of PPP loans including with respect to forgiveness of Second Draw Loans, (xxvi) an Interim Final Rule (published March 3, 2021) amending loan amount calculation for sole proprietors and independent contractors and eligibility disqualifiers, and (xxvii) an Interim Final Rule (published March 18, 2021) implementing changes to the Paycheck Protection Program as amended by the American Rescue Plan Act, and (b) the SBA Procedural Notices: (i) Guidance on Participation Sales for Paycheck Protection Program Loans, effective April 24, 2020, (ii) Guidance on Whole Loans Sales of Paycheck Protection Program Loans (Procedural Notice 5000-20024), effective May 1, 2020, (iii) Refinance of EIDL Loans with PPP loan Proceeds and Lender Remittance of EIDL Refinance Proceeds to SBA (Procedural Notice 5000-20032), effective June 19, 2020, (iv) Extension of Authority to Guarantee Paycheck Protection Program Loans, effective July 6, 2020, (v) Procedural Notice – PPP Lender Processing Fee Payment and 1502 Reporting Process (Procedural Notice 5000-20028), effective July 13, 2020, (vi) Procedures for Lender Submission of Paycheck Protection Program Loan Forgiveness Decisions to SBA and SBA Forgiveness Loan Reviews (Procedural Notice 5000-20038), effective July 23, 2020, (vii) Paycheck Protection Program Loans and Changes of Ownership (Procedural Notice 5000-20057), effective October 2, 2020, (viii) Guidance on Modifications to SBA Forms 3506, 3507 and 750 CA (for purposes of PPP only) (Procedural Notice 5000-20074), effective January 6, 2021, (ix) Guidance on Repeal of EIDL Advance Deduction Requirement for SBA Loan Forgiveness Remittances to PPP Lenders (Procedural Notice 5000-20075), effective January 8, 2021, (x) Guidance on First Draw Paycheck Protection Program Loan Increases After Enactment of the Economic Aid Act (Procedural Notice 5000-20076), effective January 13, 2021, (xi) Guidance on PPP Borrower Resubmission of Loan Forgiveness Applications Using Form 3508S, Lender Notice Responsibilities to PPP Borrowers, and Offset of Remittances to Lenders for Lender Debts (Procedural Notice 5000-20077), effective January 15, 2021, (xii) Guidance on PPP Excess Loan Amount Errors (Procedural Notice 5000-20078), effective January 15, 2021, (xiii) Guidance on PPP Procedures for Addressing Unresolved Issues on Borrower First Draw PPP Loans (Procedural Notice 5000-20083), effective January 26, 2021, (xiv) Updated Guidance on PPP Lender Processing Fee Payment and 1502 Reporting Process (Procedural Notice 5000-20091), effective February 8, 2021, and (xv) Revised Paycheck Protection Platform Procedures for Addressing Hold Codes on First Draw PPP Loans and Compliance Check Error Messages on First Draw PPP Loans and Second Draw PPP Loans (Procedural Notice 5000-20092), effective February 10, 2021.

[iv] Plus any outstanding amount under a pre-existing EIDL made on or after January 31, 2020 and before April 3, 2020.

[v] The PPPFA provides a corresponding extension to the deferral date built into the secondary market sales provisions as well.

[vi] Under the Interim Final Rule published on May 13, 2020, if a seasonal employer received a PPP loan before the alternative criterion for determining its maximum loan amount (published on April 28, 2020) and would be eligible for a higher maximum loan amount under the alternative criterion, the lender may submit a request to the SBA to upsize and make an additional disbursement in respect of such PPP loan. The lender must have not yet submitted its initial SBA Form 1502 in respect of such PPP loan and the borrower must supply the lender with the required documentation to support the increase. All caps and limitations on PPP loan amounts apply to such an increased loan.

[vii] Defined as non-profit organization or organization otherwise subject to section 511(a)(2)(B) of the Internal Revenue Code of 1986 that is a public broadcasting entity. While not expressly stated, it should be presumed that the eligibility size standards generally applicable to PPP borrowers (e.g., a cap of 500 employees) apply to such entities.

[viii] Question 56 of the FAQ states that in case where a college or university operates or holds the license for public broadcasting stations, and the station is not a separate legal entity, the limit on the number of employees "per location" applies to the public broadcasting station itself and does not include other employees of a college or university that operates or holds the license for the station.

[ix] The CARES Act waives the affiliation rules if the borrower receives financial assistance from an SBA-licensed Small Business Investment Company (SBIC) in any amount (including, per the PPP Rules, any type of financing listed in 13 CFR 107.50, such as loans, debt with equity features, equity, and guarantees). The PPP Rules further clarify that affiliation rules are waived *even if* the borrower received investment from other non-SBIC investors.

[x] In an Interim Final Rule published on May 18, 2020 (which codifies Question 44), the SBA, recognizing the ambiguity as to the inclusion or exclusion of foreign employees in its prior guidance, provided that it will not find any borrower that applied for a PPP loan prior to May 5, 2020 to be ineligible based on the borrower's exclusion of non-U.S employees from its employee headcount if the borrower (together with its affiliates) had no more than 500 employees whose principal place of residence is in the United States. Such borrowers are not deemed to have made an inaccurate certification of eligibility solely on that basis. Under no circumstances may PPP funds be used to support non-U.S. workers or operations

[xi] The SBA's determination concerning the certification regarding the necessity of the loan request will not affect SBA's loan guarantee.

[xii] The lobbying activities prohibited are defined in section 3 of the Lobbying Disclosure Act of 1995 (2 U.S.C. 1602). "'Lobbying activities' means lobbying contacts and efforts in support of such contacts, including preparation and planning activities, research and other background work that is intended, at the time it is performed, for use in contacts, and coordination with the lobbying activities of others."

[xiii] SBA guidance clarifies that this treatment follows the computation of self-employment tax from IRS Form 1040 Schedule SE Section A line 4 and removes the "employer" share of self-employment tax, consistent with how payroll costs for employees in the partnership are determined.

[xiv] With respect to loan forgiveness, this shall only apply prospectively from the date of enactment of the American Rescue Plan.

[xv] Calculations to be confirmed by the lender (via review of supporting materials provided with the Form 3508) include (i) the amount of Cash Compensation, Non-Cash Compensation, and Compensation to Owners (claimed on Lines 1, 4, 6, 7, 8, and 9 of PPP Schedule A), (ii) the amount of Business Mortgage Interest Payments, Business Rent or Lease Payments, and Business Utility Payments (claimed on Lines 2, 3, and 4 of the PPP loan Forgiveness Calculation Form), and (iii) the calculation of payroll costs divided by 0.75% (on Line 10 of the PPP loan Forgiveness Calculation Form).

[xvi] A lender must provide the SBA as part of its a forgiveness approval or denial determination: (1) the PPP loan Forgiveness Calculation Form; (2) PPP Schedule A; and (3) the (optional) PPP Borrower Demographic Information Form.

[xvii] The Notice specifies additional limitations where the change of ownership transaction is to be financed (in whole or in part) by a 7(A) loan, including that the proceeds of the loan cannot finance any escrow account required to be set up in connection with a change of ownership of a PPP borrower as described in the Notice.

[xviii] The PPP Lender is defined in the Notice as either the initial lender under the PPP loan or the lender that is then-servicing the PPP loan (i.e., if the loan has been transferred to another lender).

[xix] In the case of funds escrowed in connection with an asset sale, the Notice specifies that the PPP lender must notify the appropriate SBA Loan Servicing Center of the location and amount of funds in the escrow account within 5 business days of sale's closing. The SBA Loan Servicing Center can be found at: https://www.sba.gov/CitrusHeightsLGPC.

[xx] This new Interim Final Rule establishes a new subpart L for 13 C.F.R. § 134, which are the rules of procedure governing cases before the SBA Office of Hearings and Appeals. This new subpart L also specifies the provisions of subpart B (the OHA's general Rules of Practices) that are applicable to appeals of SBA Loan Review Decisions under subpart L.

[xxi] This guidance is not completely clear and appears to indicate that borrower's remedy in this case is to request an SBA review with respect to any amounts for which forgiveness was denied.

[xxii] EIDL borrowers may also be subject to fraud charges (and resulting fines and imprisonment) under 18 USC § 1040, which addresses fraud in connection with major disaster or emergency benefits.

[xxiii] Note that there is a misalignment with the PPP, as "new entities" are those that were not in business between February 15, 2019 and June 30, 2019. Currently, the PPP does not provide an express mechanism for calculating loan amounts for businesses that were not in operation between June 30, 2019 and February 15, 2020.

[xxiv] The SBA has confirmed that, for purposes of the PPP, an applicant's participation in an employee stock ownership plan (ESOP) does not trigger application of the affiliation rules.

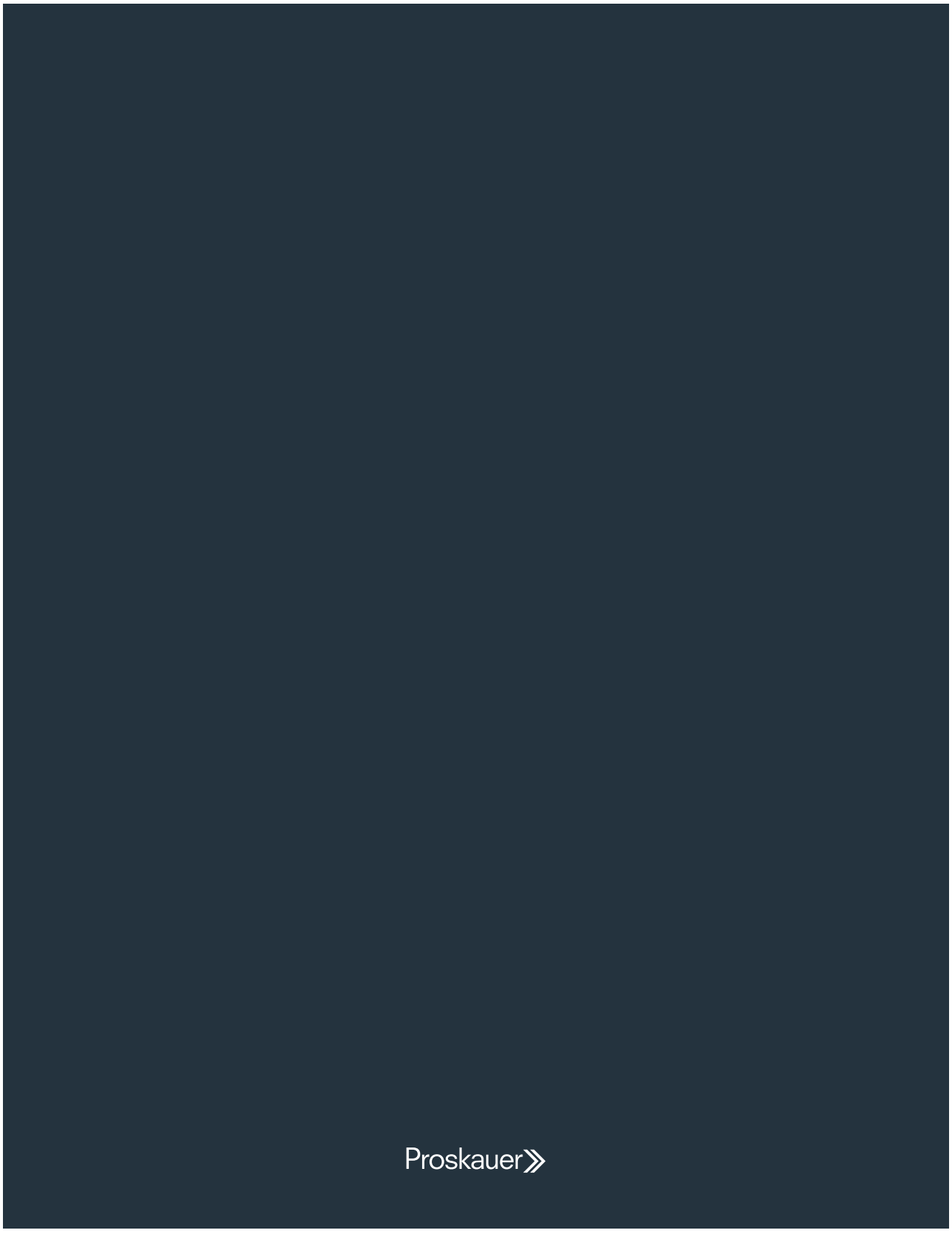
[xxv] While referred to here as PPP Lenders (as these are the institutions that ultimately lend to the end-recipients of PPP loans), the term sheet and FAQ refer to such institutions in the context of the PPPLF as PPPLF borrowers.

[xxvi] Non-recourse status of the PPPLF Loan may change if the PPP Lender breaches any of the representations, warranties, or covenants in the PPPLF documentation, or engages in fraud/misrepresentation in connection with participation in the PPPLF.

[xxvii] As described in Interim Final Rule published collectively by the Federal bank regulatory agencies (Board of Governors of the Federal Reserve System, Federal Deposit Insurance Corporation (FDIC), Office of the Comptroller of the Currency (OCC)), for participating eligible financial institutions: (i) the PPPLF is considered to be zero percent risk for purposes of risk-based and leverage-based capital requirements because PPP loans are 100% guaranteed by the SBA; and (ii) loans extended by the PPPLF to participating eligible financial institutions will not increase the regulatory capital requirements for those institutions. The Interim Final Rules take effect immediately, but are subject to a 30-day public comment period.

[xxviii] However, the Federal Reserve has clarified that this certification may be based on economic conditions in the market or markets intended to be addressed by the PPPLF facility. The certifying PPP Lender may consider current economic or market conditions as compared to usual economic or market conditions, including the availability and price of credit for small businesses with diminished revenue streams. For purposes of certifying that it is unable to secure adequate credit accommodations elsewhere, such PPP Lender does not need to establish that credit is unavailable, rather that credit accommodations may be available, but at prices or on conditions that are inconsistent with a normal, well-functioning market.

[xxix] Certain additional documentation requirements apply for depository institutions that have not already established access to the Federal Reserve's lending programs for depository institutions ("discount window" programs).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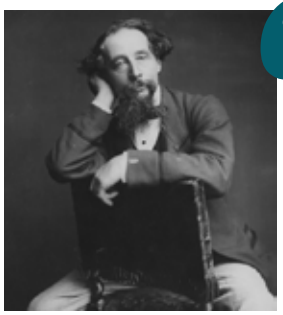
In the Gaps

Ares Alternative Credit Newsletter

Fourth Quarter 2020

Markets at the Boundary



> It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of light, it was the season of darkness, it was the spring of hope, it was the winter of despair.
>
> Charles Dickens

The social and economic context for Dickens' *A Tale of Two Cities* was a study in extremes, articulated so memorably in the book's first sentence. That opener resonates with us in a way it hasn't before as we consider market conditions over the last year.

The divergent legs of the "K" recovery, a topic we explored in last quarter's newsletter, have likewise taken on some new dimensions in recent months. Consider the following prints that we saw executed in the ABS markets recently. Lest there be any doubt, the statistics below are not typos.

> **On December 11th, 2020**, Continental Finance executed a securitization of credit card receivables to subprime borrowers. The transaction was structured with 5.5% equity subordination and a total cost of debt of 3.4%.
>
> **On January 12th, 2021**, Santander executed a securitization of auto loans to subprime borrowers. This transaction was structured with 1.6% equity subordination and a total cost of debt of 0.94%.
>
> **On February 2nd, 2021**, Toyota executed a securitization of auto leases to prime borrowers. This transaction was structured with 2.7% equity subordination and a total cost of debt of 0.27%.

As a team, we tried to remember if we had ever seen such execution in the last twenty years, such a combination of high leverage and low debt yields. No one could.

More puzzling was the level of investor demand for these bonds: every security was *multiple times oversubscribed*. It is noteworthy that these transactions are being executed in an institutional market, devoid of Reddit-induced valuation bubbles.

For whom are today's markets simultaneously the best of times and the worst of times? The answer to that question points to both risk and opportunity, and not in equal portions.

Our goal in this edition of *In the Gaps* is to explore a few of the areas in Alternative Credit where we see points of interest and pockets of potential opportunity. Relative value continues to evolve across the Alternative Credit landscape, favoring Illiquid Alternative Credit opportunities today given robust supply and strong execution in the Liquid Alternative Credit markets. In our view, current market opportunities are more idiosyncratic, less thematic. Real asset opportunities, especially opportunities to acquire large portfolios of real estate with in-place leases, became an important focus in 4Q 2020. We also saw new opportunities develop in the fund finance sector, especially out of Europe. We believe the opportunity across Alternative Credit continues to be focused on those areas where fiscal stimulus and an overabundance of liquidity have not distorted the relative value and credit picture. We anticipate a number of additional sectors and themes to emerge in 2021.

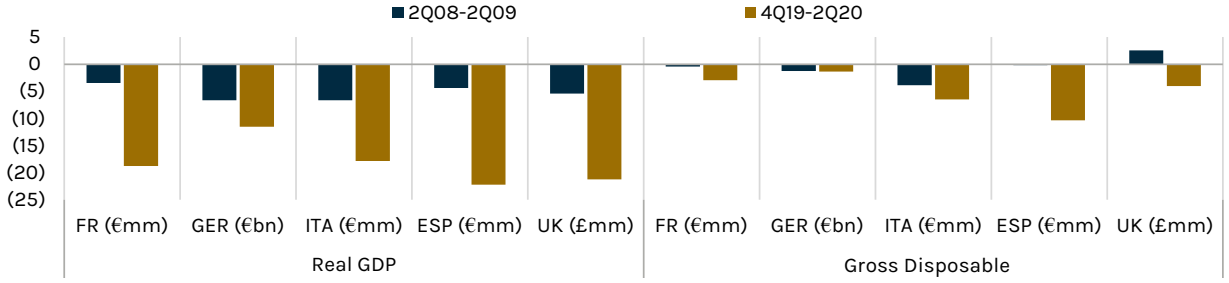
Theme: Europe

Alternative Credit in Europe is a multi-dimensional, multi-faceted set of opportunities and risks. It has always been so. Historically, much of this was driven by the uneven, country-by-country, development of credit markets. The emergence of large, institutional non-bank lenders, like Ares, over the past decade has been a large part of that development. Many markets and regions are still developing.

As it relates to the asset markets in which Ares Alternative Credit ("the Team") is active, the pandemic changed the economic and market backdrop in Europe in considerable ways. Due to the wide range of fiscal responses across various European countries and regions, asset opportunities have become very difficult for many investors to *assess* and *access* right now.

The pandemic has caused loss of employment through the closure of small businesses and employee furloughs. The economic stress across Europe has been *multiples* worse than the Global Financial Crisis ("GFC"). Exhibit 1 shows the comparative impact on GDP and household income across several European countries. Governments and central banks came to the rescue with dozens of

Exhibit 1: Real GDP & Household Income: GFC vs. Pandemic



Source: Morgan Stanley. As of June 2020.

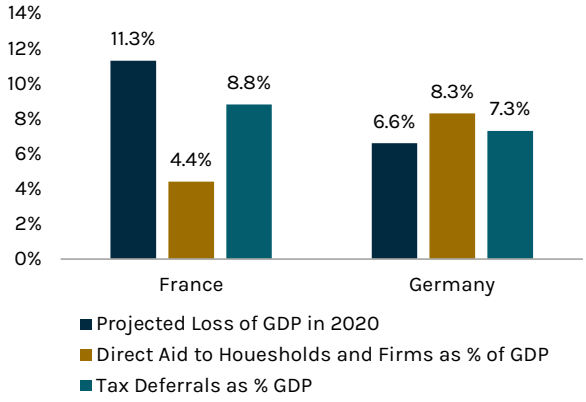
aid and economic stimulus programs, all in an effort to cushion the financial blow to households and small businesses.

Absent government stimulus and aid programs, one would have expected to see large, concomitant increases in consumer and small business loan delinquencies, defaults and ultimately losses. Each country in Europe approached the challenge of attenuating economic pain with different tools, different targets and different intensity. These differences now provide a window into the degree to which fiscal stimulus programs may mask underlying risks still unresolved.

In looking through all of the data, we thought that the experiences of Germany and France were interesting case studies. The chart "GDP Loss vs. Government Support" illustrates how the magnitude of GDP loss compares to the magnitude of the two main types of government support: direct aid and tax deferrals. Based on these figures, both countries' governments *overstimulated* their economies. In 2020, France's support had amounted to 13% of GDP compared to an 11% GDP loss, an excess of 2%. Germany's stimulus exceeded GDP loss by approximately 9% of GDP, more than double the amount of GDP loss.

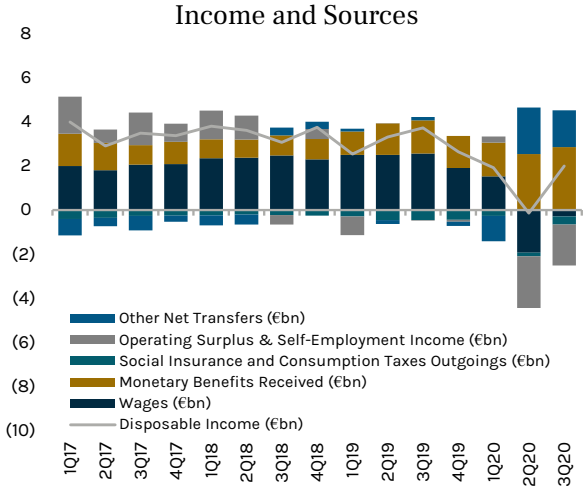
Exhibits 3 and 4 below show the overall combined impact on household income in Germany and France inclusive of each country's stimulus programs. Some of the categories differ between the two countries, but the overall picture is clear.

Exhibit 2: GDP Loss vs. Government Support



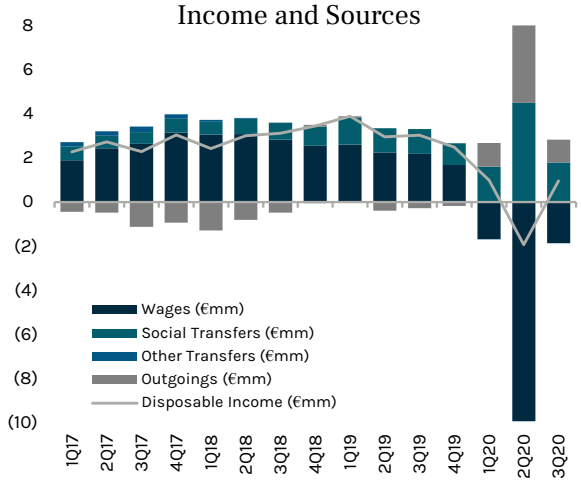
Source: IMF, OECD and Bruegel. As of October 2020.

Exhibit 3: Germany: Change in Household Income and Sources



Source: Morgan Stanley. As of September 2020.

Exhibit 4: France: Change in Household Income and Sources



Germany not only provided more government support and fiscal stimulus, but also directed a larger proportion of that aid to *individuals and households* whereas France provided relatively less support and directed a larger proportion of that aid to *businesses*. Said differently, Germany took more of a "demand side" approach where France took more of a "supply side" approach to economic stimulus. Given those different approaches, we were not surprised to see differences in consumer credit performance between the two countries. As the charts below show, delinquency rates of French consumer debt increased to a much greater degree compared to those same types of consumers and obligations in Germany.

> "These differences now provide a window into the degree to which fiscal stimulus programs may mask underlying risks still unresolved."

Exhibit 5: New Deliquency Rates – Auto Loans

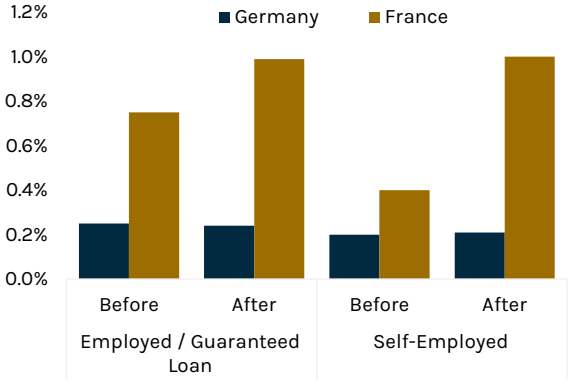
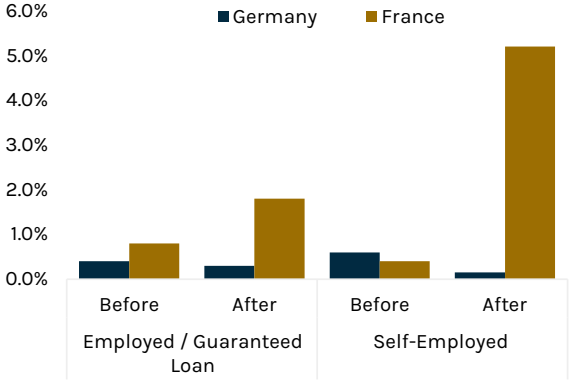


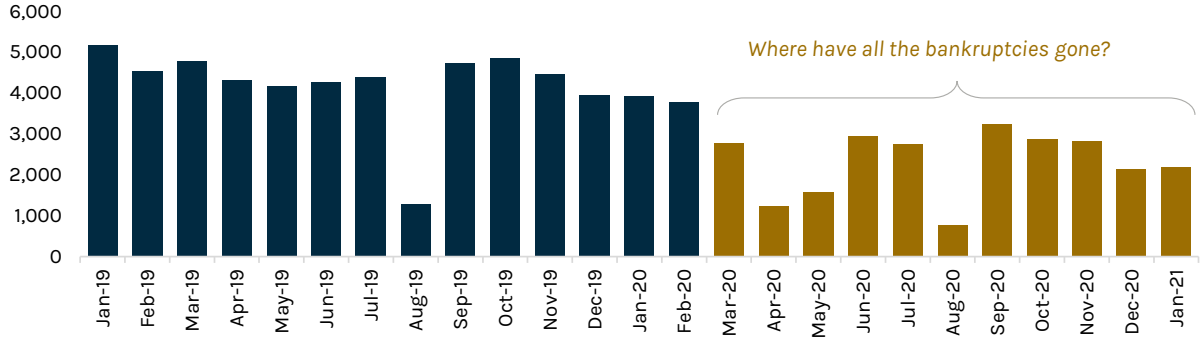
Exhibit 6: New Delinquency Rates – Unsecured Consumer Debt



Source: Goldman Sachs. As of May 2020.

We are certain that absent the significant stimulus and aid programs that occurred, the performance above would have been meaningfully worse in each case – perhaps especially so in France given the more acute drop in employment and wages. The data below plots the monthly bankruptcy filings in France. In 2019, over 51,000 French companies filed, an average of 4,252 per month. That monthly average has dropped to 2,308 amid economic distress in 2020. The drop in the number of corporate bankruptcies and insolvencies is a pattern we are seeing in many European countries. It has been affected by a number of factors including government programs and the temporary suspension of filing requirements. In many cases, these amount to simply "kicking the can down the road" – which has only postponed the day of reckoning.

Exhibit 7: Number of Monthly Corporate Bankruptcies (France)



The differences one can see between Germany and France (to say nothing of the 25 other European Union countries) highlight some of the analytical challenges that investors face today: will performance revert to the mean, or has the mean changed altogether for the next few years? What will credit performance look like once economic stimulus and support programs are withdrawn? The many and varied European markets may well provide some of the best clues into those answers.

Theme: Truth vs. Headlines

Of the many Alternative Credit sectors which drew the attention of the financial press and other commentators, perhaps no sector elicited more hyperbole than CLOs. Consider the following headlines and statements about the CLO market made last year by otherwise serious people and publications.

> "The COVID-19 pandemic is bringing an economic downturn far worse than many CLOs were designed to withstand." **Bloomberg, April 2020**
>
> "CLOs: ground zero for the next stage of the financial crisis?" **Financial Times, May 2020**
>
> "CLOs and the loans underpinning them are ground zero in terms of the vulnerability of this crisis." **UBS, May 2020**

The kernel of truth upon which these claims and speculations were based was, indeed, the inherent difficulties in assessing the breadth and depth of the pandemic's impact across many different industries. A somewhat popular, if misinformed, view heading into the pandemic was that CLO investors were somehow "exposed to greater risks than ever before," as articulated by CNBC.

In fact, CLO investors were getting exposure to the leveraged loan market in the same manner they always had. There was no excess of risks building up in CLO portfolios that left its investors suddenly exposed to risks they were uniquely bearing. As of February 28, 2020, when considering several categories of risk, the average CLO loan portfolio was remarkably similar to the broader loan market index (see Table 1).

TABLE 1

	CLO Market (Median)	CSLLI Index
Price < $90	9.1%	8.5%
2nd Lien	1.2%	3.3%
Cov Lite	87.8%	82.5%
CCC or Below	4.9%	5.3%
Retail	3.3%	3.4%
Energy	3.5%	3.2%

Source: Ares INsight database and CSLLI Index (Credit Suisse).

> "The headlines and the sensational stories claiming that CLOs were the 2020 equivalent of subprime mortgages in 2008 were just wrong."

Not only that, but many CLO portfolios were of a higher quality than the loan index. Below is a look at those same risk categories, but instead comparing the loan index to the top quartile CLO portfolio, which shows that many CLO portfolios were less risky than the broader loan market (see Table 2).

TABLE 2

	CLO Market (25th Percentile)	CSLLI Index
Price < $90	7.0%	8.5%
2nd Lien	0.6%	3.3%
Cov Lite	84.2%	82.5%
CCC or Below	3.1%	5.3%
Retail	2.2%	3.4%
Energy	2.5%	3.2%

Source: Ares INsight database and CSLLI Index (Credit Suisse).

Additionally, the broader loan market is often accessed through loan ETFs, open-end mutual funds, and other credit fund products. A comparison of the CLO structure with those vehicles again shows the advantages that the CLO structure has in times of stress (see Table 3).

TABLE 3

	CLO Structure	ETF Loan Funds
Outflows	No	Yes
Forced Selling	No	Yes
Actively Managed	Yes	No
Mark-to-Market Leverage	No	Often

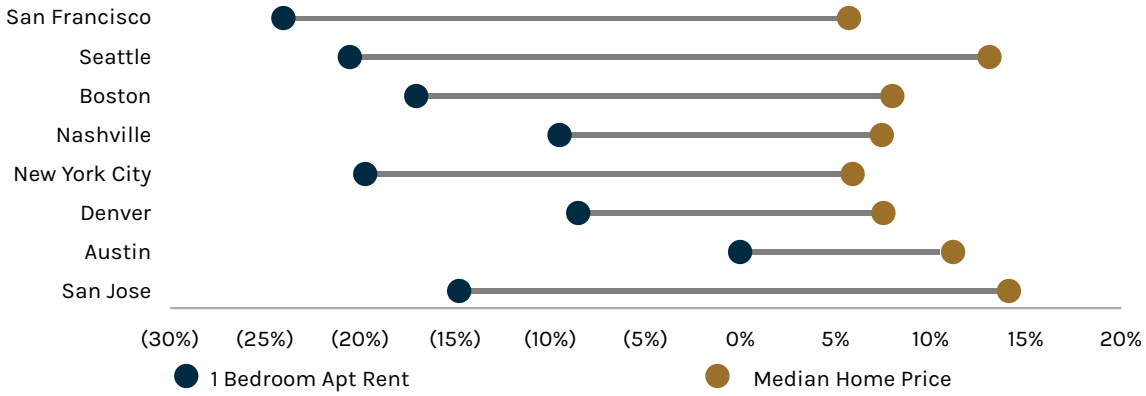
Source: Ares INsight database and CSLLI Index (Credit Suisse).

The headlines and the sensational stories claiming that CLOs were the 2020 equivalent of subprime mortgages in 2008 were just wrong. They were wrong in terms of the risk they claimed CLOs held, and the risk CLOs posed to the markets and financial system. Whether overlooked, misunderstood or intentionally ignored, the many structural advantages that CLOs possess over other forms of capital in the loan market were hiding in plain sight. Experience and data, not melodramatic headlines, offered a more constructive and informed way to approach the CLO asset class in 2020.

Inside the Data

This quarter we decided to take an updated look at the U.S. housing market. All those U-Haul truck rentals we looked at in our last letter have been relocating folks around the country. We are now starting to see the impact of net migration on apartment rents and home prices. We then look at an unexpected, year-end rally in very subordinated commercial aviation bonds and try to make sense of a market seemingly detached from fundamentals.

Exhibit 8: Percent Changes in Apartment Rent vs. Median Home Prices



Source: Zillow Data. As of December 2020.

TALES OF CITIES

In our last letter, we pointed to changes developing in the U.S. housing market. We looked through the lens of U-Haul moving truck rental rates, home inventory levels, and differences in occupancy rates between single family rentals and apartments. All of these data pointed to suburbanization trends.

This data also revealed large differences between cities and regions. Some cities were clear beneficiaries of these trends and others were clearly losing. This quarter we take another look at housing data. First we will look at changes in housing preferences within cities. Then we will compare cities of comparable size to identify trends we're seeing that point to possible other secular shifts.

Apartment living is on sale in major cities throughout the U.S. As illustrated in Exhibit 8, a number of cities where rent rates have dropped by as much as 20% or more. In these same cities, however, the price of homes has seen significant inflation. There may be a temptation to blame lower interest rates (and, by extension, lower mortgage rates) for the boom in house prices. Certainly, lower rates can help offset a higher home price through lower monthly mortgage payments. However, interest rates tell only part of the story; drivers also include recent changes in housing and lifestyle preferences accelerated by increased work-from-home flexibility.

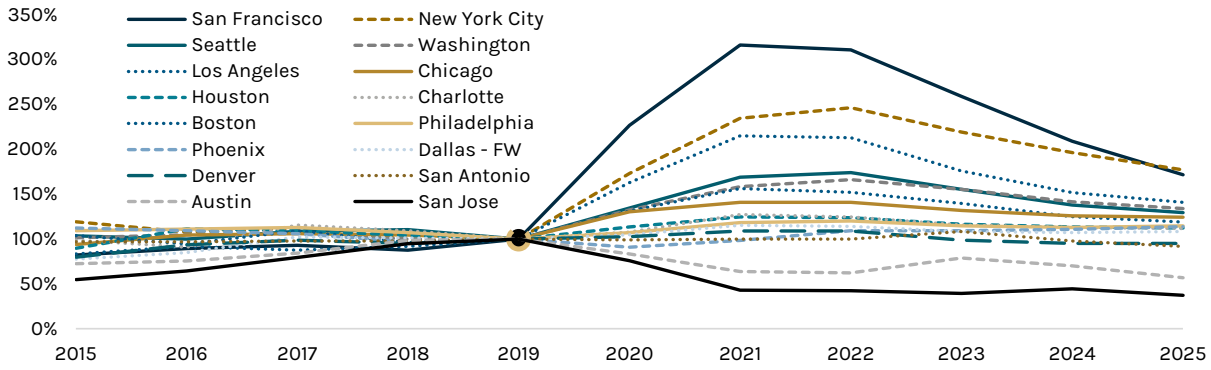
> "The top 10 performing stocks in 2020 were companies that did not have a single dollar of profit between them."

It could be a while before we see a full recovery in multi-family rental and occupancy rates driven by net migration trends. That picture will also vary considerably by city. Exhibit 9 depicts multi-family vacancy forecasts, and suggests that *peak* vacancies are still ahead of us in many cities. With only a handful of exceptions, a recovery to pre-COVID-19 vacancy levels may not occur for several more years.

VALUATION BUBBLES AND RISK AT THE BOUNDARIES

Here's a fun fact (or perhaps it's depressing): the top 10 performing stocks in 2020 were companies that did not have a single dollar of profit between them. Here's another. among the larger indices, the top five companies in terms of stock growth (Tesla, Moderna, Peloton, Zoom and Etsy) had a *combined* profit of $948 million, but grew by $967 *billion* in market value (Source: Bloomberg). However you might feel about these particular companies and their prospects, that overall picture makes us uncomfortable. We've seen this movie before a few times and the ending still manages to evoke a wince.

Exhibit 9: Change in Multi-Family Vacancies – As a Percentage of 2019 Figures



Source: CoStar. As of February 2021. Forecasts are inherently limited and should not be relied upon as indicators of actual or future outcomes.

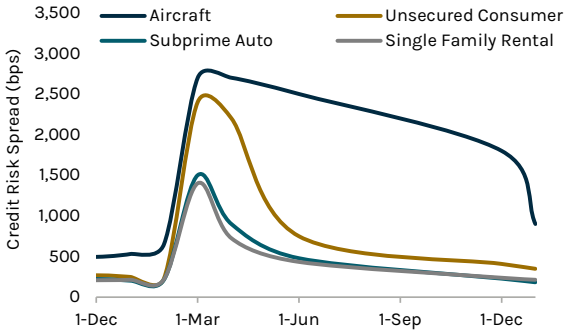
> "Multi-family vacancy forecasts suggest that *peak* vacancies are still ahead of us in many cities."

We will conclude our look *Inside the Data* with a recent development we saw within trade data across a few different public ABS sectors. While perhaps not as perplexing as the valuation swings recently witnessed in the common stock of companies like GameStop, AMC or Tesla (to say nothing of bitcoin *et al*), the data nonetheless revealed a significant increase in value based on no new information, fundamental or otherwise, so far as we can tell.

Exhibit 10 plots the market spreads observed during 2020 on double-B rated tranches in four ABS sectors, including aircraft ABS. Noting the obvious fact that commercial aviation has been impacted to an unprecedented degree in this cycle (multiples worse than previous cycles), no one was especially surprised to see the most subordinated debt tranches in aircraft ABS experience a precipitous drop in price beginning in 1Q 2020.

As the year proceeded, and with limited trading activity, prices started to recover somewhat as distressed investors hunted for an option play on a recovery in commercial aviation. That said, we have never met the person who would claim that the BB tranche of aircraft ABS is the "fulcrum security" in the capital structure today. So, it was surprising to see, based on no new information at all, these same securities rally from December to January, bringing market spreads to around 900bps (see Exhibit 10).

Exhibit 10: Select ABS Double-B Markets



Source: TRACE Data. As of December 2020.

The closest thing we have heard to an investment thesis in the wake of this dramatic move had to do with the difficulty of finding assets trading at a discount to par. It is the popular *buy convexity in a bull market* tactic. It's a tactic that can be successful where there is fundamental value and limited downside risk. Clearly, not every asset that trades at a deep discount is simply cheap. In the case of deeply subordinated aviation bonds, one has to grapple with the endemic uncertainties of aircraft values today and the risk of high loss severities due to structural leverage. The only world that would seem to justify such a valuation run is one in which fundamental risk is in the rearview mirror and fading fast.

To be clear: that is not the world we see inside the data and certainly not within the commercial aviation sector. We, across the firm, are already seeing private opportunities that validate our view that many of these public ABS trades reflect a 100% severity gamble. In our view, few things are more mispriced in the markets today than severity risk – and the siren song of convexity can be very dangerous. We do think there are opportunities brewing in the aviation sector. opportunities to acquire aircraft portfolios and to start leasing companies. Such

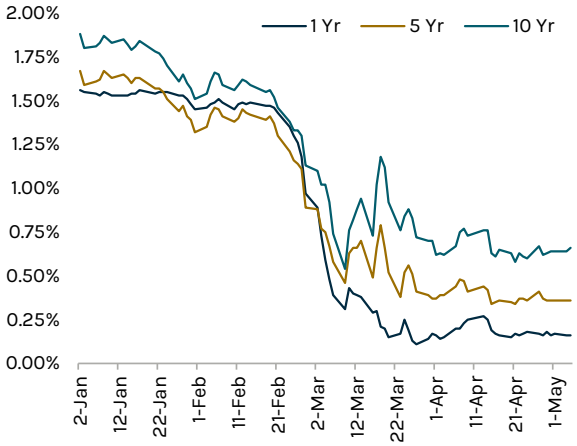
opportunities will play out over the coming years, not months driven by restructurings and other changes in that sector that are still around the corner.

> "In our view, few things are more mispriced in the markets today than severity risk…"

Theme: Mortgage Market Mayhem

It's easy to oversimplify what was a complex and wide-ranging set of conditions that impacted the mortgage market last year. That said, we think the experience of New Residential Investment Corp ("NRZ"), a publicly-traded mortgage REIT, is an informative case study. Its fortunes expose the extent to which the mortgage market was tested, and illustrates the astonishing recovery that followed.

Exhibit 11: UST Rates by Tenor



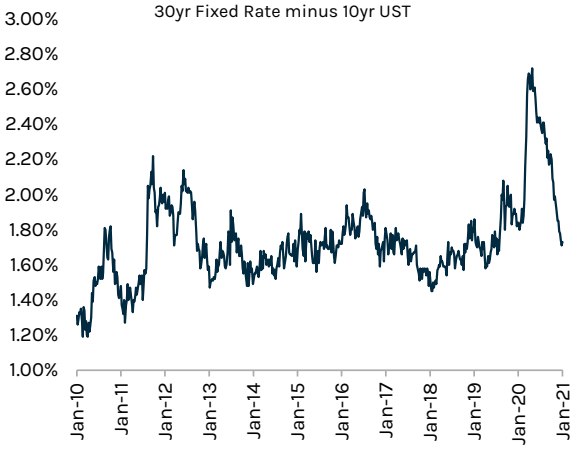
Source: Bloomberg, Federal Reserve Economic Data.
As of December 2020.

For context, in the years leading up to 2020, NRZ had been a steady performer among a cohort of steady-performing mortgage REITS. The company's stock had traded in a range between $15 and $17 per share, and right around 1.0x book value, since early 2017. Its portfolio had grown consistently by about $1.2 billion per year across its three main strategies: MSRs, Residential Mortgage Securities and Residential Mortgage Whole Loans. In 2017, NRZ acquired Shellpoint, a vertically-integrated mortgage originator and servicer. The Shellpoint platform would prove to be a most valuable asset and play a key role in NRZ's recovery.

It's hard to overstate the impact the pandemic-induced market volatility on the mortgage REIT market that occurred last March and April 2020. Liquidity pressures arising from margin calls on repo lines saw NRZ's stock price plummet to 0.28x book value by the close of business on March 24, 2020. One week later, NRZ announced a $6 billion stressed disposition of non-agency mortgages, a transaction designed to shore up the company's liquidity. On April 6, 2020, Moody's downgraded NRZ's long-term issuer rating to B3 and placed its ratings on review for further downgrade due to liquidity concerns.

On May 5, 2020, NRZ reported the impact of COVID-19 on its first quarter results which revealed just how much its business had been impacted. Later that month, the company announced that it had secured $600 million of fresh capital in the form of a three-year senior secured loan. The capital came with a relatively high cost (11% plus warrants), but also at an especially opportune time: interest rates had just collapsed by well over 100bps. Mortgage lending rates, which had been decreasing steadily since 4Q18, began to level out around 3% (see Exhibit 11). This resulted in the widest mortgage spread environments we have seen since the GFC.

Exhibit 12: Mortgage Spreads



Source: Bloomberg, Federal Reserve Economic Data.
As of December 2020.

With the backdrop of high origination volumes, it was clear when NRZ released second quarter earnings on July 22, 2020 that the tide had started to turn. In fact, just three months later the company announced that it had prepaid its 11% secured term loan with proceeds from a five-year, unsecured note with a 6.25% coupon and that it had issued eight new securitizations representing $3.5 billion of mortgage collateral.

As year-end approached, NRZ provided further insight into the astonishing extent of its recovery. The company, which had generated pre-tax income of $58 million in 2018, reported generating $934 million for FY20. Compared

to 2018, origination volumes were up nearly 9x and its servicing portfolio had nearly tripled – achievements made possible by the company's integrated mortgage originator and servicing platform (i.e., Shellpoint).

NRZ was hardly alone among mortgage REITs in experiencing such a swing in fortunes. Compressed, as it was, into mere months, NRZ's experience is something of a time-lapse video preview of similar swings that are occurring in other Alternative Credit sectors, albeit at a much slower pace.

We have spoken at length about the issues that asset owners and originators frequently face even in normal times: access to liquidity; capital constraints; barriers to traditional markets; the need for flexible capital solutions. These issues have always contributed to the Alternative Credit market opportunity. However, following such a shock as was experienced in 2020, these are the defining issues and continue to be the major drivers of opportunity in Alternative Credit.

Path Forward

2020 will be a year that will be remembered for a long time. Movies will be made, books will be written, and history will be memorialized on the causes, the impact, and the recovery from our first global pandemic in almost 100 years. As we think about market conditions today and what they might suggest about the future, let's return for a moment to where we started.

We described today's market conditions as the *best of times and the worst of times*. As fitting as that metaphor is today, it is almost certainly not how this market will be remembered. When we look back a decade from now, a very different narrative will be told that will show a more complete picture of the impact of 2020. The ultimate global recovery and its impact on life expectancy, demographics and economics will play out over the next several years. The market has not yet dealt with the losses that have been created from the events of 2020.

The ultimate winners and losers of the post-pandemic economy have not been determined. If someone suggests that they know how all of this will play out, even just the rest of 2021, you should question anything else they are telling you. From our vantage point in the asset markets, signs and signals continue to flash caution. Valuations in many corners continue to diverge from fundamentals (a topic we visited in our 2Q 2020 *In the Gaps* newsletter).

By way of update, we welcomed two new members to the Team in 4Q 2020, Moureen Karim and Nate Kim. Ms. Karim serves as an Associate on the Alternative Credit Product Management and Investor Relations Team, joining from the Public Finance Investment Banking Housing Group at Citigroup. Previously, Ms. Karim worked in the Citigroup Markets Program as an Analyst in the Structured Credit Sales and Equity Derivatives Sales Groups. Mr. Kim joined as an Associate and portfolio analyst. Previously, Mr. Kim was a member of the Complex Securities Valuation Group at Ernst & Young, where he specialized in the valuation of complex financial securities including equity and fixed income derivatives, preferred and common stocks, convertible debt and preferred, loan portfolios and contingent consideration arrangements for financial reporting, corporate strategy, mergers and acquisitions transaction support and regulatory compliance purposes.

Speaking of team, some of you might know that Alternative Credit's mantra is "*Kaizen Investing with Purpose*" an expression of our commitment to improvement as investors. An important element in that continuous improvement process is regular introspection. At the end of every year, the Team receives a holiday homework project consisting of answering 10-15 questions designed to provoke self-reflection on a personal and professional level. We ask our Team members to take time to step back and gain perspective on a number of topics, identifying their most important lessons learned. Given the extraordinary events of 2020, and everyone's first pandemic experience, we conducted this *Lessons Learned* process twice. Below is a selection of some of those lessons learned we wanted to share.

- "Portfolios and diversity allow you to move forward with certainty in uncertain times."
- "Risk doesn't disappear but it can be shifted or transformed into other risks (like squeezing a long balloon)."
- "You can't control what others do and how they behave. If you aren't with good people, move on until you find some. Then stick to them like glue."
- "*Man moves forward.* I stole this from Will Durant from *Lessons from History* but it seems so applicable this year. No matter how bad the headlines or the circumstances, never bet against humans' ability to adapt, overcome and move forward. It actually gives me hope for climate change and demographic headwinds. Never bet against adaptation."
- "The key to a successful team is the genuine shared feeling of wanting each member of your team to succeed."

We thank you again for your continued interest and support of the Team and our purpose, and look forward to our upcoming *In the Gaps* webinar on Wednesday, March 17th, 2021 from 9:00 – 10:15am EDT. Please look for more details in your email.

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Charity Spotlight of the Quarter

Malaria Consortium

Ares is committed to investing in global health and education to help save lives and drive equality. Ares' Alternative Credit Flagship Fund will donate at least 10% of the fund's carried interest to global health and education charities. Given Ares' focus on investing with purpose, each quarter we will highlight a non-profit organization with a track record of delivering value per charitable dollar contributed. Note Ares is not endorsing the non-profit organization, nor has Ares donated to the highlighted charity at the time of this publication.

COMBATING COMMUNICABLE DISEASES IN CHILDREN

Founded in 2003, Malaria Consortium is a non-profit organization that specializes in the prevention, control and treatment of malaria and other communicable diseases among vulnerable populations.

Malaria kills over 400,000 people annually, >90% of them are children under five in sub-Saharan Africa. Typically, the disease is transmitted through infected mosquitos and involves flu like symptoms including fever. Along with death, anemia and other ailments, it is believed that malaria can also cause permanent disabilities such as hearing impairment, visual impairment or epilepsy. Malaria Consortium's seasonal malaria chemoprevention (SMC) program provides preventative antimalarial medication to children under five in the Sahel region of Africa.

Using training and supervision materials that it has developed, Malaria Consortium enables governments and health care officials to distribute sulfadoxine-pyrimethamine (SP) and amodiaquine (AQ) medication to those in need. They are involved with logistical and technical operations which start with procuring the medicine, shipping it to state and national level warehouses and last mile distribution to their over 100,000 health facilities.

Once at the health facilities, their network of over 100,000 community distributers – almost all of whom are volunteers that come from the communities they serve – disperse the medication with over a >92% coverage for their target demographics. The community distributers are trained in determining eligibility, how to check for side effects and how to refer children who might already have malaria directly to local health systems.

Community distributers go door-to-door within their geographies to distribute medication and teach caregivers how to mix and administer the medication for the subsequent doses (2 over the next 3 days). The medication lasts one month, so the processes is completed once a month during the rainy season.

EVIDENCE OF EFFECTIVENESS:

- In clinical trials, SMC has found to prevent 75% of malaria cases in children under five
- Research has found that it is feasible to implement high coverage SMC at scale using existing national health systems
- Serious adverse drug reactions have proven to be rare
- The economic cost of administering four monthly cycles of SMC was estimated to be $3.63 per child

SMC needs to be well accepted by beneficences to ensure maximum uptake. It's vital that communities understand the rationale behind SMC and support its implementation. This typically involves sensitization meetings with local leaders, radio spots and town announcers disseminating relevant information during campaigns.

During their campaigns, the community distributers collect monitoring data which are aided by Malaria Consortium tools to better track disbursement and effectiveness. They also conduct surveys using LQAS methodology to rapidly assess coverage in target areas and identify room to improvement.

Malaria Consortium has been a leading implementer of SMC since WHO issued its recommendation to scale up the intervention in 2012. It began with a pilot in Nigeria in 2013, then rapidly scaling through the Achieving Catalytic Expansion of Seasonal Malaria Chemoprevention in the Sahel (ACESS-SMC) project in 2015-2017 in which close to seven million children were reached in Burkina Faso, Chad, Guinea, Mali, Niger, Nigeria and The Gambia. In 2021, Malaria Consortium aims to reach 15.8m children across 5 different countries and 25 states/regions.

NOTABLE FACTS:

- 95% of Malaria Consortium staff are located in malaria endemic areas
- Malaria Consortium currently operates 12 projects in countries across Africa and Southeast Asia
- In 2018, Burkina Faso's total population was 19.8 million and there were an estimated 7.9 million malaria cases with 13,000 deaths
- A total of 31 million children are currently eligible for SMC
- In 2020, an estimated 895,000 eligible children were not covered – which is a $4.4 million funding gap. In 2019, an estimated 9.1 million eligible children were not covered
- If eligibility broadens, funding needs will increase significantly

Going forward, Malaria Consortium hopes to expand further than its current countries and begin distributing SPAQ into countries with unconfirmed evidence of effectiveness due to differences in transmission seasons and weather. In 2021, they are targeting 2 districts in the Nampula province of Mozambique (co-funded by the Bill and Melinda Gates Foundation) and 2 districts in Karamoja. They expect to be conducting research to explore the feasibility, acceptability and impact of SMC. As these projects continue and if viability is proven, Malaria Consortium's target population will be rapidly expanded, requiring precise planning and additional funding.

Away from SMC, Malaria Consortium is conducting research in a few interesting development areas including AI managed propagation to separate male mosquitos for sterilization, surveillance of malaria parasite genomic traits to aid in resistance development and digitally supported community health systems which would expand beyond malaria.

> "SMC is a great success story and we must not just preserve it, but nurture it and adapt it."
>
> **Dr. Pedro Alonso**
> *Director of the WHO Global Malaria Programme*

QUICK FACTS ON MALARIA CONSORTIUM'S FINANCIALS

- ✓ In 2019, Malaria Consortium spent ~$20m towards distributing SMC
- ✓ Charity assessment organization GiveWell estimates that all in costs for Malaria Consortium to save a child's life is $3,000-$5,000 to save a child's life
- ✓ GiveWell estimates that Malaria Consortium could absorb up to $4.7m for work in 2022-2023
- ✓ Partners include Open Philanthropy Project, Global Fund, UNICEF and the Bill & Melinda Gates Foundation

FOR ADDITIONAL INFORMATION, PLEASE VISIT WWW.MALARIACONSORTIUM.ORG.

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Low Interest Rates, Market Power, and Productivity Growth*

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Abstract

This study provides a new theoretical result that a decline in the long-term interest rate can trigger a stronger investment response by market leaders relative to market followers, thereby leading to more concentrated markets, higher profits, and lower aggregate productivity growth. This strategic effect of lower interest rates on market concentration implies that aggregate productivity growth declines as the interest rate approaches zero. The framework is relevant for anti-trust policy in a low interest rate environment, and it provides a unified explanation for rising market concentration and falling productivity growth as interest rates in the economy have fallen to extremely low levels.

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Interest rates have fallen to extreme lows across advanced economies, and they are projected to stay low. At the same time, market concentration, business profits, and markups have been rising steadily. The rise in concentration has been associated with a substantial decline in productivity growth; furthermore, the productivity gap between leaders and followers within the same industry has risen. This study investigates the effect of a decline in interest rates on investments in productivity enhancement when firms engage in dynamic strategic competition. The results suggest that these broad secular trends—declining interest rates, rising market concentration, and falling productivity growth—are closely linked.

In traditional models, lower interest rates boost the present value of future cash flows associated with higher productivity, and therefore lower interest rates encourage firms to invest in productivity enhancement. This study highlights a second strategic force that reduces aggregate investment in productivity growth at very low interest rates. When firms engage in strategic behavior, market leaders have a stronger investment response to lower interest rates relative to followers, and this stronger investment response leads to more market concentration and eventually lower productivity growth.

The model is rooted in the dynamic competition literature (e.g. Aghion et al. (2001)). Two firms compete in an industry both intra-temporally, through price competition, and inter-temporally, by investing in productivity-enhancing technology. Investment increases the probability that a firm improves its productivity position relative to its competitor. The decision to invest is a function of the current productivity gap between the leader and the follower, which is the state variable in the industry. A larger productivity gap gives the leader a larger share of industry profits, thereby making the industry more concentrated. The model includes a continuum of industries, all of which feature the dynamic game between a leader and follower. Investment decisions within each industry induce a steady state stationary distribution of productivity gaps across markets and hence overall industry concentration and productivity growth.

The theoretical analysis is focused on the following question: What happens to aggregate investment in productivity enhancement when the interest rate used to discount profits falls? The model's solution includes the "traditional effect" through which a decline in the interest rate leads to more investment by market leaders and market followers. However, the solution to the model also reveals a "strategic effect" through which market leaders invest more aggressively relative to market followers when interest rates fall. The central theoretical result of the analysis shows that the strategic effect dominates the traditional effect at a sufficiently low interest rate; as the interest rate approaches zero, it is guaranteed that economy-wide measures of market concentration will rise and aggregate

productivity growth will fall.

The intuition behind the strategic effect can be seen through careful consideration of the investment responses of market leaders and market followers when the interest rate falls. When the interest rate is low, the present value of a persistent market leader becomes extremely high. The attraction of becoming a persistent leader generates fierce and costly competition especially if the two firms are close to one another in the productivity space. When making optimal investment decisions, both market leaders and market followers realize that their opponent will fight hard when their distance closes. However, they respond asymmetrically to this realization when deciding how much to invest. Market leaders invest more aggressively in an attempt to ensure they avoid neck-and-neck competition. Market followers, understanding that the market leader will fight harder when they get closer, become discouraged and therefore invest less aggressively. The realization that competition will become more vicious and costly if the leader and follower become closer in the productivity space discourages the follower while encouraging the leader. The main proposition shows that this strategic effect dominates as the interest rate approaches zero.

The dominance of the strategic effect at low interest rates is a robust theoretical result. This result is shown first in a simple example that captures the basic insight, and then in a richer model that includes a large state space and hence richer strategic considerations by firms. The existence of this strategic effect and its dominance as interest rates approach zero rests on one key realistic assumption, that technological catch-up by market followers is gradual. That is, market followers cannot "leapfrog" the market leader in the productivity space and instead have to catch up one step at a time. This feature provides an incentive for market leaders to invest not only to reach for higher profits but also to endogenously accumulate a strategic advantage and consolidate their leads. This incentive is consistent with the observations that real-world market leaders may conduct defensive R&D, erect entry barriers, or engage in predatory acquisition as in Cunningham et al. (2019). This assumption is also supported by the fact that gradual technological advancement is the norm in most industries, especially in recent years (e.g., Bloom et al. (2020)).

The exploration of the supply side of the economy is embedded into a general equilibrium framework to explore whether the mechanism is able to quantitatively account for the decline in productivity growth. The general equilibrium analysis follows the literature in assuming that the long-run decline in interest rates is generated by factors on the demand side of the economy, which is modeled as a reduction in the discount rate of households. We conduct a simple calibration of the model and show that the model

generates a quantitatively meaningful rise in the profit share and decline in productivity growth following the decline in the interest rate from 1984 to 2016 in the United States.

The insights from the model have implications for anti-trust policy. Policies that tax leader profits or subsidize follower investments are less effective than one that dynamically facilitates technological advancements of followers. Furthermore, more aggressive anti-trust policy is needed during times of low interest rates. The baseline model abstracts from financial frictions by assuming that market leaders and market followers face the same interest rate. We believe that the introduction of financial frictions that generate a gap between the interest rates faced by market leaders and market followers would lead to even stronger leader dominance when interest rates fall.[1] For example, using data on interest rates and imputed debt capacity, we show that the decline in long-term rates has disproportionately favored industry leaders relative to industry followers.

The model developed here is rooted in dynamic patent race models (e.g. Budd et al. (1993)). These models are notoriously difficult to analyze; earlier work relies on numerical methods (e.g. Budd et al. (1993), Acemoglu and Akcigit (2012)) or imposes significant restrictions on the state space to keep the analysis tractable (e.g. Aghion et al. (2001) and Aghion et al. (2005)). We bring a new methodology to this literature by analytically solving for the recursive value functions when the discount rate is small. This new technique enables us to provide sharp, analytical characterizations of the asymptotic equilibrium as discounting tends to zero, even as the ergodic state space becomes infinitely large. The technique should be applicable to other stochastic games of strategic interactions with a large state space and low discounting.

This study also contributes to the large literature on endogenous growth.[2] The key difference between the model here and other studies in the literature, e.g. Aghion et al. (2001) and Peters (forthcoming), is the assumption that followers have to catch up to the leader gradually and step-by-step instead of being able to close all gaps at once. This assumption provides an incentive for market leaders to accumulate a strategic advantage, which is a key strategic decision that is relevant in the real world. We show this key "no-leapfrog" feature overturns the traditional intuition that low interest rates always promote investment, R&D, and growth; instead, when interest rates are sufficiently low, this strategic effect always dominates the traditional effect, and aggregate investment and productivity growth will fall.

[1] For related work on financial constraints and productivity growth, see Caballero et al. (2008), Gopinath et al. (2017) and Aghion et al. (2019a).

[2] Recent contributions to this literature include Acemoglu and Akcigit (2012), Akcigit et al. (2015), Akcigit and Kerr (2018), Cabral (2018), Garcia-Macia et al. (2018), Acemoglu et al. (forthcoming), Aghion et al. (forthcoming), and Atkeson and Burstein (forthcoming), among others.

In contemporaneous work, Akcigit and Ates (2019) and Aghion et al. (2019b) respectively argue that a decline in technology diffusion from leaders to followers and the advancement in information and communication technology—which enables more efficient firms to expand—could have contributed to the rise in firm inequality and low growth. While we do not explicitly study these factors in the model here, the economic forces highlighted by our theory suggest that low interest rates could magnify market leaders' incentives to take advantage of these changes in the economic environment. More broadly, the theoretical result of this study suggests that the literature exploring the various reasons behind rising market concentration and declining productivity growth should consider the role of low interest rates in contributing to these patterns.

This paper is also related to the broader discussion surrounding "secular stagnation" in the aftermath of the Great Recession. Some explanations, e.g., Summers (2014), focus primarily on the demand side and highlight frictions such as the zero lower bound and nominal rigidities.[3] Others such as Barro (2016) have focused more on the supply-side, arguing that the fall in productivity growth is an important factor in explaining the slow recovery. This study suggests that these two views might be complementary. For example, the decline in long-term interest rates might initially be driven by a weakness on the demand side. But a decline in interest rates can then have a contractionary effect on the supply-side by increasing market concentration and reducing productivity growth. An additional advantage of this framework is that one does not need to rely on financial frictions, liquidity traps, nominal rigidities, or a zero lower bound to explain the persistent growth slowdown such as the one we have witnessed since the Great Recession.

1 Motivating Evidence

Existing research points to four secular trends in advanced economies that motivate the model. First, there has been a secular decline in interest rates across almost all advanced economies. Rachel and Smith (2015) show a decline in real interest rates across advanced economies of 450 basis points from 1985 to 2015. The nominal 10-year Treasury rate has declined further from 2.7% in January 2019 to 0.6% in July 2020. This motivates the consideration of extremely low interest rates on firm incentives to invest in productivity enhancement.

Second, measures of market concentration and market power have risen substantially over this same time frame. Rising market power can be seen in rising markups (e.g., Hall

[3]See e.g., Krugman (1998), Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), Benigno and Fornaro (forthcoming), and Eggertsson et al. (2019).

(2018), De Loecker et al. (2020), Autor et al. (2020)), higher profits (e.g., Barkai (forthcoming), De Loecker et al. (2020)), and higher concentration in product markets (e.g., Grullon et al. (2019), Autor et al. (2020)). Diez et al. (2019) from the International Monetary Fund put together a firm-level cross-country dataset from 2000 onward to show a series of robust facts across advanced economies. Measures of markups, profitability, and concentration have all risen. They also show that the rise in markups has been concentrated in the top 10% of firms in the overall markup distribution, which are firms that have over 80% of market share in terms of revenue.

Third, productivity growth has stalled across advanced economies (e.g., Cette et al. (2016), Byrne et al. (2016)). It is important to note that this slowdown in productivity began before the Great Recession, as shown convincingly in Cette et al. (2016). The slowdown in productivity growth has been widespread, and was not initiated by the Great Recession.

Fourth, the decline in productivity growth has been associated with a widening productivity gap between leaders and followers and reduced dynamism in who becomes a leader. Andrews et al. (2016b) show that the slowdown in global productivity growth is associated with an expanding productivity gap between "frontier" and "laggard" firms. In addition, the study shows that industries in which the productivity gap between the leader and the follower is rising the most are the same industries where sector-aggregate productivity is falling the most.

Berlingieri et al. (2017) use firm level productivity data from OECD countries to estimate productivity separately for "leaders," defined as firms in the 90th percentile of the labor productivity distribution for a given 2-digit industry, and "followers," defined as firms in the 10th percentile of the distribution. The study shows that the gap between leaders and followers increased steadily from 2000 to 2014. Both the Andrews et al. (2016b) and Berlingieri et al. (2017) studies point to the importance of the interaction between market leaders and market followers in understanding why productivity growth has fallen over time. Andrews et al. (2016a) show that the tendency for leaders, which they call frontier firms, to remain market leaders has increased substantially from the 2001 to 2003 period to the 2011 to 2013 period. They conclude that it has become harder for market followers to successfully replace market leaders over time.

As shown below, these facts are consistent with the model's prediction of what happens when interest rates fall to low levels. Furthermore, even the timing of these patterns is consistent with the results of the model. As shown below, the model predicts that market power increases as the interest rate declines, but productivity growth has an inverted-U relationship and only declines when the interest rate becomes sufficiently low. In the real-world, the decline in real interest rates began in the 1980s, and measures of

market concentration began rising in the late 1990s. The trends in productivity growth, in contrast, began later. Most studies place the beginning of the period of a decline in productivity growth between 2000 and 2005, and the rising productivity gap between market leaders and followers also emerged at this time.

2 A Stylized Example

Declining interest rates in advanced economies have been associated with a rise in market power, a widening of productivity and markups between market leaders and followers, and a decline in productivity growth. This section begins the theoretical analysis of the effect of a decline in interest rates on market concentration and productivity growth. More specifically, we begin by presenting a stylized example to illustrate the key force in the model: low interest rates boost the incentive to invest for industry leaders more than for industry followers. Section 3 below presents the full model.

Consider two firms competing in an industry. Time is continuous and as in the dynamic patent race literature, there is a technolgoical ladder such that firms that are further ahead on the ladder are more productive and earn higher profits. The distance between two firms on the technological ladder represents the state variable for an industry. To keep the analysis as simple as possible, we assume that an industry has only three states: firms can compete neck-and-neck (state=0) with flow profit $\pi_0 = 1/2$ each, they can be one step apart earning flow profits $\pi_1 = 1$ for the leader and $\pi_{-1} = 0$ for the follower, or they can be two steps apart. If firms are two steps apart, that state becomes permanent, with leader and follower earning $\pi_2 = 1$ and $\pi_{-2} = 0$ perpetually.

Firms compete by investing at the rate η in technology in order to out-run the other firm on the technological ladder. The firm pays a flow investment cost $c(\eta) = -\eta^2/2$ and advances one step ahead on the technological ladder with Poisson rate η. Starting with a technological gap of one step, if the current follower succeeds before the leader, their technological gap closes to zero. The two firms then compete neck-and-neck, both earning flow profit $1/2$ and continue to invest in order to move ahead on the technological ladder. Ultimately each firm is trying to get two steps ahead of the other firm in order to enjoy permanent profit of $\pi_2 = 1$.

Given the model structure, we can solve for equilibrium investment levels. At an interest rate r, the value of a permanent leader is $v_2 \equiv 1/r$ and the value of a permanent follower is $v_{-2} \equiv 0$. Firms that are zero or one-step apart choose investment levels to maximize their firm values, taking the other firm's investment level as given. The equilibrium

firm value functions satisfy the following HJB equations:

$$rv_1 = \max_{\eta} \pi_1 - \eta^2/2 + \eta\,(v_2 - v_1) + \eta_{-1}\,(v_0 - v_1) \tag{1}$$

$$rv_0 = \max_{\eta} \pi_0 - \eta^2/2 + \eta\,(v_1 - v_0) + \eta_{0}\,(v_{-1} - v_0) \tag{2}$$

$$rv_{-1} = \max_{\eta} \pi_{-1} - \eta^2/2 + \eta\,(v_0 - v_{-1}) + \eta_{1}\,(v_{-2} - v_{-1}) \tag{3}$$

where $\{\eta_{-1}, \eta_0, \eta_1\}$ denote the investment choices in equilibrium.

The intuition behind the HJB equations can be understood using equation (1) that relates the flow value rv_1 for a one-step-ahead leader to its three components: flow profits minus investment costs $(\pi_1 - \eta^2/2)$, a gain in firm value of $(v_2 - v_1)$ with Poisson rate η if the firm successfully innovates, and a loss in firm value of $(v_0 - v_{-1})$ with Poisson rate η_{-1} if the firm's competitor successfully innovates.

Both firms compete dynamically for future profits and try to escape competition in order to enjoy high profits π_2 indefinitely. Suppose the industry is in state 1. Then the investment intensity for the leader and the follower are given by the first order conditions from HJB equations, $\eta_1 = v_2 - v_1$ and $\eta_{-1} = v_0 - v_{-1}$, respectively. Intuitively, the magnitude of investment effort depends on the slope of the value function for the leader and the follower. The follower gains value from reaching state=0 so it has a chance to become the leader in the future; the leader gains value from reaching to state=2 not because of higher flow profits (note $\pi_1 = \pi_2 = 1$) but, importantly, by turning its temporary leadership into a permanent one.

The key question is, what happens to equilibrium investment efforts in state 1 if there is a fall in interest rate r? The answer, summarized in proposition 1, is that the leader's investment η_1 rises by more than the follower's investment η_{-1} as r falls. In fact, as $r \to 0$, the difference between leader's and follower's investment diverges to infinity.

Proposition 1. *A fall in the interest rate r raises the market leader's investment more than it raises the follower's, and their investment gap goes to infinity as r goes to zero. Formally, $d\eta_1/dr < d\eta_{-1}/dr$ with $\lim_{r\to 0}(\eta_1 - \eta_{-1}) = \infty$.*

All proofs are in the appendix. The intuition for $(\eta_1 - \eta_{-1}) \to \infty$ is as follows. Since $\eta_1 = v_2 - v_1$, a fall in r increases investment for the leader as the present value of its monopoly profits ($v_2 \equiv 1/r$) is higher were it to successfully innovate. However, for the follower $\eta_{-1} = v_0 - v_{-1}$, and the gain from a fall in r is not as high due to the *endogenous* response of its competitor in state=0 were the follower to successfully innovate. In particular, a fall in r also makes firms compete *more fiercely* in the neck-and-neck state

zero. A fall in r thus increases the expectation of a tougher fight were the follower to successfully catch up to $s = 0$. While the expectation of a more fierce competition in future state-zero disincentivizes the follower from catching up, the possibility of "escaping" the fierce competition through investment raises the incentive for the leader. This strategic asymmetry continues to amplify as $r \to 0$, giving us the result.

The core intuition in this example does not depend on simplifying assumptions such as exogenous flow profits, quadratic investment cost, state independent investment cost, or limiting ourselves to three states. For example, the full model that follows allows for an infinite number of possible states, microfounded flow profits with Bertrand competition, investment cost advantage for the follower, and other extensions. Also note that we do not impose any financing disadvantage for the follower vis-a-vis the leader as they both face the same cost of capital r. Any additional cost of financing for the follower, as is typically the case in practice, is likely to further strengthen our core result.

The key assumption for the core result is that follower cannot "leapfrog" the leader. As we explained, the key intuition relies on the expectation that the follower will have to "duke it out" in an intermediate state (state zero in our example) before it can get ahead of the leader. This expectation creates the key strategic asymmetry between the response by the leader and the follower to a lower interest rate. The follower is discouraged by the fierce competition in the future if it were to successfully close the technological gap between itself and the leader. The same is not true for the leader. In fact for the leader in state $s = 1$, the expectation of more severe competition in state $s = 0$ makes the leader want to escape competition with even greater intent. All of this gives the leader a larger reward for investment relative to the follower as r falls. We discuss the plausibility and applicability of the no-leapfrogging assumption in more detail below, and we show that the idea of incremental innovation applies to a wide range of settings in the real world.

The next section moves to a more general setting with a potentially infinite number of states. This breaks the rather artificial restriction of the simple example that leadership becomes perpetual in state 2. In the general set up, the leader can continue to create distance between itself and the follower by investing, but it cannot guarantee permanent leadership. Adding this more realistic dimension to the framework brings out additional important insights: not only does a fall in r increase the investment gap between the leader and the follower, but for r low enough, the average follower stops investing all together thereby killing competition in the industry. Therefore, while the example imposed permanent leadership exogenously, the full model shows that leadership endogenously becomes permanent. And as in the example, the expectation of permanent leadership makes the temporary leader invest more aggressively in a low interest rate environment.

As a result, a fall in the interest rate to a very low level raises market concentration and profits, and ultimately reduces productivity growth.

3 Model

The model has a continuum of markets with each market having two firms that compete with each other for market leadership. Firms compete along a technological ladder where each step of the ladder represents productivity enhancement. The number of steps, or states, is no longer bounded, so firms can move apart indefinitely. Firms' transition along the productivity ladder is characterized by a Poisson process determined by the level of investment made by each firm.

We aggregate across all markets and define a stationary distribution of market structures and the aggregate productivity growth rate. Section 4 characterizes the equilibrium and analyzes how market dynamism, aggregate investment, and productivity growth evolve as the interest rate declines toward zero. This section and Section 4 evaluate the model in partial equilibrium taking the interest rate and the income of the consumer as exogenously given. Section 5 then endogenizes these objects by embedding the model into general equilibrium.

3.1 Consumer Preferences

Time is continuous. At each instance t, a representative consumer decides how to allocate one unit of income across a continuum of duopoly markets indexed by v, maximizing

$$\max_{\{y_1(t;\nu),y_2(t;\nu)\}} \exp\left\{\int_0^1 \ln\left[y_1\left(t;\nu\right)^{\frac{\sigma-1}{\sigma}}+y_2\left(t;\nu\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} d\nu\right\} \tag{4}$$

$$\text{s.t.} \int_0^1 p_1\left(t;\nu\right) y_1\left(t;\nu\right)+p_2\left(t;\nu\right) y_2\left(t;\nu\right) d\nu = 1,$$

where $y_i\left(t;\nu\right)$ is the quantity produced by firm i of market v and $p_i\left(t;\nu\right)$ its price. The consumer preferences in (4) is a Cobb-Douglas aggregator across markets ν, nesting a CES aggregator with elasticity of substitution $\sigma > 1$ across the two varieties within each market.

Let $P\left(t\right) \equiv \exp\left(\int_0^1 \ln\left[p_1\left(t;\nu\right)^{1-\sigma}+p_2\left(t;\nu\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}\right)$ be the consumer price index. Cobb-Douglas preferences imply that total revenue of each market is always one, i.e. $p_1\left(t;\nu\right) y_1\left(t;\nu\right)+p_2\left(t;\nu\right) y_2\left(t;\nu\right) = 1$. Hence firm-level sales only depend on the

relative prices within each market and are independent of prices in other markets, i.e. $\frac{y_1(t;\nu)}{y_2(t;\nu)} = \left(\frac{p_1(t;\nu)}{p_2(t;\nu)}\right)^{-\sigma}$. This implies that all strategic considerations on the firm side take place within a market and are invariant to prices outside a given market.

3.2 Firms: Pricing and Investment Decisions

The two firms in a market are indexed by $i \in \{1, 2\}$ and we drop the market index ν to avoid notational clutter. Each firm has productivity z_i with unit cost of production equal to λ^{-z_i} for $\lambda > 1$. Given consumer demand described earlier, each firm engages in Bertrand competition to solve,

$$\max_{p_i} \left(p_i - \lambda^{-z_i}\right) y_i \quad \text{s.t. } p_1 y_1 + p_2 y_2 = 1 \text{ and } y_1/y_2 = (p_1/p_2)^{-\sigma}. \tag{5}$$

The solution to this problem can be written in terms of state variable $s = |z_1 - z_2| \in \mathbb{Z}_{\geq 0}$ that captures the productivity gap between the two firms. When $s = 0$, two firms are said to be neck-and-neck; when $s > 0$, one firm is a temporary leader while the other is a follower. Let π_s denote leader's profit in a market with productivity gap s, and likewise let π_{-s} be the follower's profit of the follower in the market. Conditioning on the state variable s, firm profits π_s and π_{-s} no longer depend on the time index or individual productivities and have the following properties.

Lemma 1. *Given productivity gap s, the solution to Bertrand competition leads to flow profits*

$$\pi_s = \frac{\rho_s^{1-\sigma}}{\sigma + \rho_s^{1-\sigma}}, \qquad \pi_{-s} = \frac{1}{\sigma\rho_s^{1-\sigma} + 1},$$

where ρ_s defined implicitly by $\rho_s^\sigma = \lambda^{-s}\frac{\left(\sigma\rho_s^{\sigma-1}+1\right)}{\sigma+\rho_s^{\sigma-1}}$ is the relative price between the leader and the follower. Equilibrium markups are $m_s = \frac{\sigma+\rho_s^{1-\sigma}}{\sigma-1}$ and $m_{-s} = \frac{\sigma\rho_s^{1-\sigma}+1}{(\sigma-1)\rho_s^{1-\sigma}}$.

Lemma 2. *Under Bertrand competition, follower's flow profit π_{-s} is weakly-decreasing and convex in s; leader's and joint profits, π_s and $(\pi_s + \pi_{-s})$, are bounded, weakly-increasing, and eventually concave in s.*[4] *Moreover, $\lim_{s\to\infty} \pi_s > \pi_0 \geq \lim_{s\to\infty} \pi_{-s}$.*

Lemma 2 states that a higher productivity gap is associated with higher profits for the leader and for the market as a whole. We therefore interpret markets in a lower state to be more competitive than markets in a higher state. Markups are also (weakly) increasing in the state s.

[4] A sequence $\{a_s\}$ is eventually concave iff there exists $\bar{s}$ such that a_s is concave in s for all $s \geq \bar{s}$.

Our main theoretical results hold under any sequence of flow profits $\{\pi_s\}_{s=-\infty}^{\infty}$ that satisfy the properties in Lemma 2, as our proofs show. Such a profit sequence could be generated by alternative forms of competition (e.g., Cournot) or anti-trust policies (e.g., constraints on markups or taxes on profits). For clarity, even though $\lim_{s\to\infty}\pi_s = 1$ under Bertrand, we let $\pi_\infty \equiv \lim_{s\to\infty}\pi_s$ denote the limiting profit of an infinitely-ahead leader, and we derive our theory using the notation π_∞.

As an example under Bertrand competition, when duopolists produce perfect substitutes ($\sigma \to \infty$), profits are $\pi_s = 1 - e^{-\lambda s}$ for leaders and $\pi_{-s} = 0$ for followers and neck-and-neck firms. As another example outside of the Bertrand microfoundation, our main results hold for the following sequence of profits: $\pi_s = 0$ if $s < 1$ and $\pi_s = \pi_\infty > 0$ if $s \geq 1$, i.e. all leaders receive the identical flow profits whereas followers and neck-and-neck firms have zero profit.

Investment Choice

The most important choice in the model is the investment decision of firms competing for market leadership. A firm that is currently in the leadership position incurs investment cost $c(\eta_s)$ in exchange for Poisson rate η_s to improve its productivity by one step and lower the unit cost of production by a factor of $1/\lambda$. The corresponding follower firm chooses its own investment η_{-s} and state s transitions over time interval Δ according to,

$$s(t+\Delta) = \begin{cases} s(t)+1 & \text{with probability } \Delta\cdot\eta_s, \\ s(t)-1 & \text{with probability } \Delta\cdot(\kappa+\eta_{-s}), \\ s(t) & \text{otherwise.} \end{cases}$$

where parameter $\kappa \geq 0$ is the exogenous catch-up rate for the follower. There is a natural catch-up advantage that the follower enjoys due to technological diffusion from the leader to the follower; this guarantees the existence of a non-degenerate steady-state and is a standard feature in patent-race-based growth models (e.g., Aghion et al. (2001), and Acemoglu and Akcigit (2012)).

Firms discount future payoffs at interest rate r which is taken to be exogenous from the perspective of firm decision-making.[5] Firm value $v_s(t)$ equals the expected present-

[5] We illustrate in Section 5.1 how r is endogenously determined in general equilibrium and can be viewed as coming from the household discount rate.

discount-value of future profits net of investment costs:

$$v_s(t) = \mathbb{E}\left[\int_0^\infty e^{-r\tau}\left\{\pi(t+\tau) - c(t+\tau)\right\} d\tau \middle| s\right]. \tag{6}$$

Value function (6) illustrates the various incentives that collectively determine how a firm invests. The basic problem is not only inter-temporal, but most importantly, *strategic*. A firm bears the investment cost today but obtains the likelihood of enhancing its market position by one-step which earns it higher profits in the future. However, there is also an important strategic dimension embedded in (6), as a firm's expected gain from investment today is also implicitly a function of how its competitor is expected to behave in the future. For instance, in the example of Section 2, intensified competition in the neck-and-neck state has a discouragement effect on the follower's investment and a motivating effect on the leader's.

We impose regularity conditions on the cost function $c(\cdot)$ so that firm's investment problem is well-defined and does not induce degenerate solutions. Specifically, we assume $c(\cdot)$ is twice continuously differentiable and weakly convex over a compact investment space: $c'(\eta_s) \geq 0$, $c''(\eta_s) \geq 0$ for $\eta_s \in [0, \eta]$. We assume the investment space is sufficiently large, $c(\eta) > \pi_\infty$ and $\eta > \kappa$—so that firms can compete intensely if they choose to—and $c'(0)$ is not prohibitively high relative to the gains from becoming a leader ($c'(0)\kappa < \pi_\infty - \pi_0$)—otherwise no firm has any incentive to ever invest.

We look for a stationary Markov-perfect equilibrium such that the value functions and investment decisions are time invariant and depend only on the state. The HJB equations for firms in state $s \geq 1$ are

$$rv_s = \pi_s + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \max_{\eta_s \in [0,\eta]} \left[\eta_s (v_{s+1} - v_s) - c(\eta_s)\right] \tag{7}$$

$$\begin{aligned} rv_{-s} = \; & \pi_{-s} + \eta_s \left(v_{-(s+1)} - v_{-s}\right) + \kappa \left(v_{-(s-1)} - v_{-s}\right) \\ & + \max_{\eta_{-s} \in [0,\eta]} \left[\eta_{-s} \left(v_{-(s-1)} - v_{-s}\right) - c(\eta_{-s})\right]. \end{aligned} \tag{8}$$

In state zero, the HJB equation for either market participant is

$$rv_0 = \pi_0 + \eta_0 (v_{-1} - v_0) + \max_{\eta_0 \in [0,\eta]} \left[\eta_0 (v_1 - v_0) - c(\eta_0)\right]. \tag{9}$$

These HJB equations have the same intuition as those in equations (1) through (3) in our earlier example. The flow value in state s is composed of current profit net of investment

cost, capital gain from successfully advancing on the technological ladder, and capital loss if the firm is pushed back on the ladder.

Definition 1. (**Equilibrium**) Given interest rate r, a symmetric Markov-perfect equilibrium is an infinite collection of value functions and investments $\{v_s, v_{-s}, \eta_s, \eta_{-s}\}_{s=0}^{\infty}$ that satisfy equations (7) through (9). The collection of flow profits $\{\pi_s, \pi_{-s}\}_{s=0}^{\infty}$ is generated by Bertrand competition as in Lemma 1.

3.3 Aggregation Across Markets: Steady State and Productivity Growth

The state variable in each market follows an endogenous Markov process with transition rates governed by investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$ of market participants. We define a steady-state equilibrium as one in which the distribution of productivity gaps in the entire economy, $\{\mu_s\}_{s=0}^{\infty}$, is time invariant. The steady-state distribution of productivity gaps must satisfy the property that, over each time instance, the density of markets leaving and entering each state must be equal.

Definition 2. (**Steady-State**) Given equilibrium investment $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$, a steady-state is the distribution $\{\mu_s\}_{s=0}^{\infty}$ ($\sum \mu_s = 1$) over the state space that satisfies:

$$\underbrace{2\mu_0\eta_0}_{\substack{\text{density of markets}\\ \text{going from state 0 to 1}}} = \underbrace{(\eta_{-1}+\kappa)\,\mu_1}_{\substack{\text{density of markets}\\ \text{going from state 1 to 0}}}, \tag{10}$$

$$\underbrace{\mu_s\eta_s}_{\substack{\text{density of markets}\\ \text{going from state s to s+1}}} = \underbrace{\left(\eta_{-(s+1)}+\kappa\right)\mu_{s+1}}_{\substack{\text{density of markets}\\ \text{going from state s+1 to s}}} \quad \text{for all } s>0. \tag{11}$$

where the number "2" in equation (10) reflects the fact that a market leaves state zero if either firm's productivity improves.

We define aggregate productivity $Z(t)$ as the inverse of the total production cost per unit of the consumption aggregator:

$$\lambda^{Z(t)} \equiv \frac{\exp\left(\int_0^1 \ln\left[y_1(t;\nu)^{\frac{\sigma-1}{\sigma}} + y_2(t;\nu)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)}{\int_0^1 \lambda^{-z_1(t;\nu)} y_1(t;\nu) + \lambda^{-z_2(t;\nu)} y_2(t;\nu)\,d\nu}, \tag{12}$$

where recall λ is the step size of productivity increments. Note that $\lambda^{-Z(t)}$ is also the ideal cost index for the nested CES demand system in (4).

The next lemma characterizes the steady-state productivity growth rate as a function of the steady-state distribution in productivity gaps $\{\mu_s\}_{s=0}^{\infty}$ and firm-level investments $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$.

Lemma 3. *In a steady state, the aggregate productivity growth rate* $g \equiv \frac{d \ln \lambda^{Z(t)}}{dt}$ *is*

$$\begin{aligned} g &= \ln \lambda \cdot \left(\sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0 \right) \\ &= \ln \lambda \cdot \sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa \right). \end{aligned}$$

The productivity gap distribution is stationary in a steady state and, on average, the productivity growth rate at the frontier—leaders and neck-and-neck firms—is the same as that of market followers. Consequently, Lemma 3 states that aggregate productivity growth g is equal to the average rate of productivity improvements for leaders and neck-and-neck firms, weighted by the fraction of markets in each state (first equality), and that g can be equivalently written as the average rate of productivity improvements for market followers (second equality).

4 Analytical Solution

4.1 Linear Cost Function

The dynamic game between the two firms is complex and has rich strategic interactions, with potentially infinite state-contingent investment levels by each player to keep track of. To achieve analytical tractability, throughout this section we assume the cost function is linear in investment intensity: $c(\eta_s) = c \cdot \eta_s$ for $\eta_s \in [0, \eta]$. The model with a convex cost function is solved numerically in Section 5, where we show that the core results carry through. Because of linearity, firms generically invest at either the upper or lower bound in any state; hence, investment effectively becomes a binary decision, and any interior investments can be interpreted as firms playing mixed strategies. For expositional ease, we focus on pure-strategy equilibria in which $\eta_s \in \{0, \eta\}$, but all formal statements apply to mixed-strategy equilibria as well.

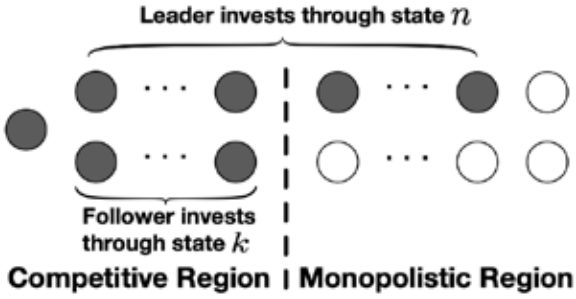
4.2 Market Equilibrium

The rest of this section solves for the equilibrium investment decisions by the leader and the follower in a market, dropping the market index ν for brevity. When the interest rate is prohibitively high, there may be a trivial equilibrium in which even the neck-and-neck firms in state zero do not invest, and aggregate investment and productivity growth are both zero in the steady state. Because the main result evaluates the effect of low interest rates r, for expositional simplicity we restrict analysis to equilibria with positive investment in the neck-and-neck state, and we present results that hold across all non-trivial equilibria.

Let $n+1 \equiv \min\{s|\eta_s < \eta\}$ be the first state in which the market leader does not strictly prefer to invest, and likewise, let $k+1 \equiv \min\{s|\eta_{-s} < \eta\}$ be the first state in which the market follower does not strictly prefer to invest.

Lemma 4. *In any non-trivial equilibrium, the leader invests in more states than the follower, $n \geq k$. Moreover, the follower does not invest ($\eta_{-s} = 0$) in states $s = k+2, ..., n+1$.*

Figure 1: Illustration of Equilibrium Structure



Lemma 4 establishes that the leader must maintain investment in (weakly) more states than the follower does. The structure of an equilibrium can thus be represented by Figure 1. States are represented by circles, going from state 0 on the left to state $n+1$ on the right. The coloring of a circle represents investment decisions: states in which the firm invests are represented by dark circles, whereas white ones represent those in which the firm does not invest. The top row represents leaders' investment decisions while the bottom row represents followers'. The corresponding steady-state features positive mass of markets in states $\{0, 1, \ldots, n+1\}$, and we can partition the set of non-neck-and-neck states into two regions: one in which the follower invests ($\{1, \ldots, k\}$) and the other in which the follower does not ($\{k+1, \ldots, n+1\}$). In the first region, the productivity gap widens

with Poisson rate η and narrows with rate $(\eta + \kappa)$. In expectation, the state s tends to decrease in this region, and the market structure tends to become more competitive. For this reason, we refer to this as the *competitive* region. Note this label does not reflect competitive market conduct or low flow profits—leaders' profits can still be high in this region—instead, the label reflects the fact that joint profits tend to decrease over time. In the second region, the downward state transition occurs at a lower rate (κ), and the market structure tends to stay or become more monopolistic and concentrated. We refer to this as the *monopolistic* region.

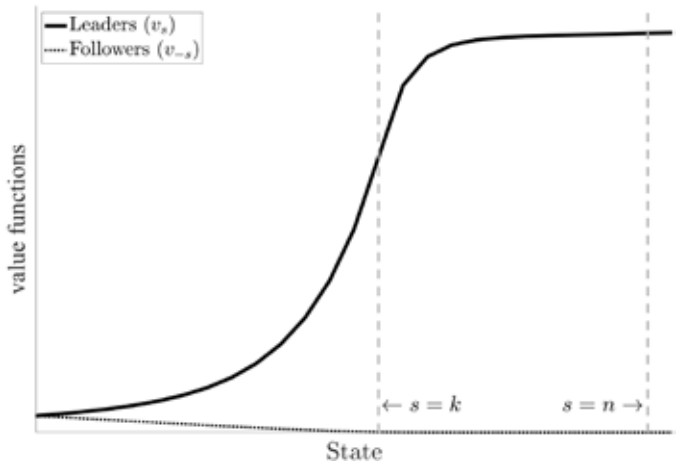
The formal proof of Lemma 4 is in the appendix; the intuition behind the $n \geq k$ proof is as follows. Suppose the leader stops investing before the follower does, $n < k$. In this case, the high flow payoff π_{n+1} is transient for the leader and the market leadership of being $n+1$ steps ahead is fleeting, because the follower invests in state $n+1$ and the rate of downward state transition is high $(\eta + \kappa)$. This implies a relatively low upper bound on the value for the leader in state $n+1$. However, because firms are forward-looking and their value functions depend on future payoffs, the low value in state $n+1$ "trickles down" to value functions in all states, meaning the incentive for the follower to invest—motivated by the future prospect of eventually becoming the leader in state $n+1$—is low. This generates a contradiction to the presumption that follower invests in more states than the leader does.

Figure 2 shows the value functions for both the leader and the follower, which help explain their investment decisions. The solid black curve represents the value function of the leader, whereas the dotted black curve represents the value function of the follower. The two dashed and gray vertical lines respectively represent k and n, the last states in which the follower and the leader invest, respectively.

The firm value in any state is a weighted average of the flow payoff in that state and the firm value in neighboring states, with weights being functions of the Poisson rate of state transitions.[6] Figure 2 shows that $v_0 - v_{-k}$ is substantially lower than $v_k - v_0$; in fact, the joint value of both firms is strictly increasing in the state: $2v_0 < v_1 + v_{-1} < \cdots < v_{n+1} + v_{-(n+1)}$. This is due to three complementary forces. First, joint profits $(\pi_s + \pi_{-s})$ are increasing in the state (Lemma 2). Second, as both firms invest in the competitive region, their investment cost further lowers the flow payoffs in the competitive region relative to the later, monopolistic region, i.e., states $k+1$ through $n+1$. Third, again because both players invest in the competitive region, a firm close to state 0 expects having to incur investment costs for a substantial amount of time before it will be able to escape

[6]For instance, for s in the competitive region ($0 < s < k$), $v_s = \frac{\pi_s - c\eta_s + \eta_s v_{s+1} + (\eta_{-s} + \kappa) v_{s-1}}{r + \eta_s + \eta_{-s} + \kappa}$, as implied by equation (7).

Figure 2: Value functions



the region and move beyond state $k + 1$.

The inequalities $2v_0 < v_s + v_{-s} < v_n + v_{-n}$ hold for any $s < n$ and imply that the leader's incentive to invest and move from state 0 to s is always higher than the follower's incentive to move from state $-s$ to 0 (as $v_s - v_0 > v_0 - v_{-s}$). Likewise, the leader's incentive to move from state s to n is always higher than follower's incentive to move from state $-n$ to $-s$. The valuation difference $v_0 - v_{-s}$ is low precisely because both firms compete intensely in states 0 through k, and their investment costs dissipate future rents. This is the sense in which strategic competition serves as a deterrent to the follower. The fact that competition serves as an endogenous motivator to the leader for racing ahead manifests itself through the convexity of the value function of a leader in the competitive region. As the leader approaches the end of the competitive region ($s = k$), its value function increases sharply, as maintaining its leadership would become substantially easier once the leader escapes the competitive region and gets to the monopolistic region. Conversely, falling back is especially costly to a leader within reach of the monopolistic region, precisely because of the intensified competition when $s < k$.

Why does the leader continue to invest in states $k + 1$ through n, even though the follower does not invest in those states? It does so to consolidate its strategic advantage. Because of technological diffusion κ, leadership is never guaranteed to be permanent, and a leader always has the possibility of falling back. As the value of being a far-ahead leader is substantially higher than being in the competitive region—due to intense competition in states 0 through k—it is worthwhile for the leader to create a "buffer" between its current state and the competitive region. The further ahead is the leader, the longer it expects to

stay in the monopolistic region before falling back to state k.

For sufficiently large s, both firms cease to invest. This happens to the follower because it is too far behind—its firm value is low, and the marginal value of catching up by one step is not worth the investment cost. This is known as the "discouragement effect" in the dynamic contest literature (Konrad (2012)). The leader eventually ceases investment as well, due to a "lazy monopolist" effect: the "buffer" has diminishing value, and once the lead $(n-k)$ is sufficiently large, an additional step of security is no longer worth the investment costs.

4.3 Steady State

The steady-state of an equilibrium can be characterized by the investment cutoff states, n and k.[7] The aggregate productivity growth rate in the steady-state is a weighted average of the productivity growth rate in each market; hence, aggregate growth depends on both the investment decisions in each state as well as the stationary distribution over states, which in turn is a function of the investment decisions. Given the investment cutoffs (n,k), equations (10) and (11) enable us to solve for the stationary distribution $\{\mu_s\}$ in closed form. The following result builds on Lemma 3 and shows that the aggregate growth rate can be succinctly summarized by the fraction of markets in the competitive and monopolistic regions.

Lemma 5. *In a steady-state induced by equilibrium investment cutoffs (n,k), the aggregate productivity growth rate is*

$$g = \ln \lambda \left(\mu^C \cdot (\eta + \kappa) + \mu^M \cdot \kappa \right),$$

where $\mu^C \equiv \sum_{s=1}^{k} \mu_s$ is the fraction of markets in the competitive region and $\mu^M \equiv \sum_{s=k+1}^{n+1} \mu_s$ is the fraction of markets in the monopolistic region. The fraction of markets in each region

[7] Technically, because we do not assume leader profits $\{\pi_s\}$ are always concave, the leader may resume investment after state $n+1$. However, because market leaders do not invest in state $n+1$, the investment decisions beyond state $n+1$ are irrelevant for characterizing the steady-state because there are no markets in those states. Moreover, because $\{\pi_s\}$ is eventually concave in s (c.f. Lemma 2), all equilibria follow a monotone structure when interest rate r is small.

satisfies

$$\mu_0 + \mu^C + \mu^M = 1, \quad \mu_0 \propto (\kappa/\eta)^{n-k+1}(1+\kappa/\eta)^k/2,$$
$$\mu^C \propto (\kappa/\eta)^{n-k}\left((1+\kappa/\eta)^k - 1\right), \quad \mu^M \propto \frac{1-(\kappa/\eta)^{n-k+1}}{1-\kappa/\eta}.$$

Lemma 5 follows from the fact that aggregate productivity growth is equal to the average rate of productivity improvements for market followers (Lemma 3). It shows the fractions of markets in the competitive and monopolistic regions are sufficient statistics for steady-state growth, and that markets in the competitive region contribute more to aggregate growth than those in the monopolistic region. Intuitively, both firms invest in the competitive region, and, consequently, productivity improvements are rapid, the state transition rate is high, dynamic competition is fierce, leadership is contentious, and market power tends to decrease over time. On the other hand, the follower ceases to invest in the monopolistic region, and, once markets are in this region, they tend to become more monopolistic over time. The monopolistic region also includes state $n+1$, where even the leader stops investing. On average, this region features a low rate of state transition and low productivity growth.

Equilibrium investment cutoffs (n,k) affect aggregate growth through their impact on the fraction of markets in each region. Lemma 5 implies that holding n constant, a higher k always draws more markets into the competitive region, thereby raising the steady-state productivity growth rate. On the other hand, holding $k \geq 1$ constant—that followers invest at all—a higher n reduces productivity growth by expanding the monopolistic region and reducing the fraction of markets in the competitive region. We formalize this discussion into a Corollary, and we further provide lower bounds for the steady-state investment and growth rate when $k \geq 1$.

Corollary 1. *Consider an equilibrium with investment cutoffs (n,k). The steady-state growth rate g is always increasing in k, and g is decreasing in n if and only if $k \geq 1$.*

Lemma 6. *Consider an equilibrium with investment cutoffs (n,k). If $k \geq 1$, then in a steady-state, the aggregate investment is bounded below by $c \cdot \kappa$, and the productivity growth rate is bounded below by $\ln\lambda \cdot \kappa$.*

4.4 Comparative Steady-State: Declining Interest Rates

The key theoretical results of the model concern the limiting behavior of aggregate steady-state variables as the interest rate declines toward zero. Conventional intuition suggests

that, when firms discount future profits at a lower rate, the incentive to invest should increase because the cost of investment declines relative to future benefits. This intuition holds in our model, and we formalize it into the following lemma.

Lemma 7. $\lim_{r\to 0} k = \lim_{r\to 0} (n-k) = \infty$.

The result suggests that, as the interest rate declines, firms in all states tend to raise investment. In the limit as $r \to 0$, firms sustain investment even when arbitrarily far behind or ahead: followers are less easily discouraged, and leaders are less lazy.

However, the fact that firms raise investment in all states does not translate into high aggregate investment and growth. These aggregate variables are averages of the investment and productivity growth rates in each market, weighted by the steady-state distribution. A decline in the interest rate not only affects the investment decisions in each state but also shifts the steady-state distribution. As Lemma 5 shows, a decline in the interest rate can boost aggregate productivity growth if and only if it expands the fraction of markets in the competitive region; conversely, if more markets are in the monopolistic region—for instance if n increases at a "faster" rate than k—aggregate productivity growth rate could slow down, as Corollary 1 suggests.

Our main result establishes that, as $r \to 0$, a slow down in aggregate productivity growth is inevitable and is accompanied by a decline in investment and a rise in market power.

Theorem 1. *As $r \to 0$, aggregate productivity growth slows down:*

$$\lim_{r\to 0} g = \ln \lambda \cdot \kappa.$$

In addition,

1. *No markets are in the competitive region, and all markets are in the monopolistic region:*
$$\lim_{r\to 0} \mu^C = 0; \qquad \lim_{r\to 0} \mu^M = 1.$$
2. *The productivity gap between leaders and followers diverges:*
$$\lim_{r\to 0} \sum_{s=0}^{\infty} \mu_s s = \infty.$$

3. *Aggregate investment to output ratio declines:*

$$\lim_{r\to 0} c \cdot \sum_{s=0}^{\infty} \mu_s \left(\eta_s + \eta_{-s}\right) = c\kappa.$$

4. *Leaders take over the entire market, with high profit shares and markups:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} \mu_s \pi_s = \pi_\infty.$$

Under Bertrand competition, the average sales of market leaders converges to 1 and that of followers converges to zero; aggregate labor share in production converges to zero.

5. *Market dynamism declines, and leadership becomes permanently persistent:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} M_s \mu_s = \infty,$$

where M_s is the expected time before a leader in state s reaches state zero.

6. *Relative market valuation of leaders and followers diverges:*

$$\lim_{r\to 0} \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.$$

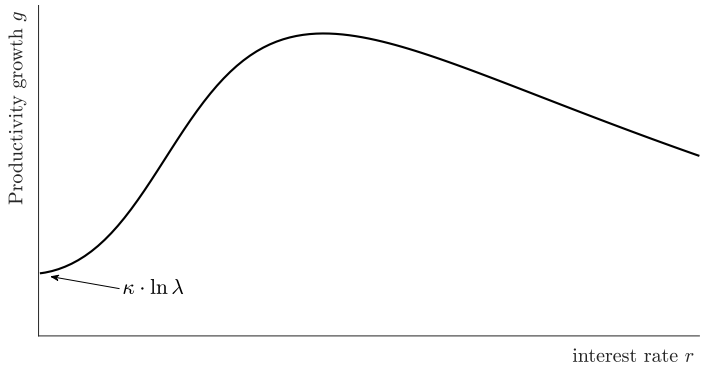
The Theorem states that, as $r \to 0$, all markets in a steady-state are in the monopolistic region, and leaders almost surely stay permanently as leaders. Followers cease to invest completely, and leaders invest only to counteract technology diffusion κ. As a result, aggregate investment and productivity growth decline and converge to their respective lower bounds governed by the parameter κ.

In the model, a low interest rate affects steady-state growth through two competing forces. As in traditional models, a lower rate is expansionary, as firms in all states tend to invest more (Lemma 7). On the other hand, a low rate is also anti-competitive, as the leader's investment response to a decline in r is stronger than follower's response. This anti-competitive force changes the distribution of market structure toward greater market power, thereby reducing aggregate investment and productivity growth. Theorem 1 shows that the second force always dominates when the level of the interest rate r is sufficiently low.

In fact, the limiting rate of productivity growth, $\kappa \cdot \ln \lambda$, is independent of the limiting profit $\lim_{s\to\infty} \pi_s$ and the investment cost c. Theorem 1 therefore has precise implications for anti-trust policies. As we elaborate in Section 6, policies that raise κ can promote growth, whereas policies that reduce leader profits or reduce the follower's investment costs are ineffective when r is low.

Because $\kappa \ln \lambda$ is the lower bound on productivity growth (c.f. Lemma 6), Theorem 1 implies an inverted-U relationship between steady-state growth and the interest rate, as depicted in Figure 3. In a high-r steady-state, few firms invest in any markets, and aggregate productivity growth is low. A marginally lower r raises all firms' investments, and the expansionary effect dominates. When the interest rate is too low, however, most markets are in the monopolistic region, in which followers cease to invest, and aggregate productivity growth is again low. The anti-competitive effect of a low interest rate also generates other implications: the leader-follower productivity gap widens, the relative leader-follower market valuation diverges, the profit share and markups rise, and business dynamism declines.

Figure 3: Steady-state growth and the interest rate: inverted-U



Lemma 7 shows that, as $r \to 0$, the *number* of states in both the competitive and monopolistic regions grow to infinity, but n and k may grow at different rates. Theorem 1 shows that the *fraction* of markets in the monopolistic region μ^M converges to one, which can happen only if the monopolistic region expands at a "faster rate" than the competitive region, i.e., the leader raises investment "faster" in response to a low r than the follower does. In the Appendix, we provide a sharp characterization on the exact rate of divergence for k and $(n-k)$ and the rate of convergence for $\mu^M \to 1$ (Lemma A.4).

To understand the leader's stronger investment response, we again turn to Figure 2. The shape of the value functions in the figure holds for any r. As the figure demonstrates,

the leader's value close to state 0 in the competitive region is small relative to its value in state $n+1$, and the leader would experience a sharp decline in value if it slips back from the monopolistic region into the competitive region. When the interest rate is low, a patient leader invests even far into the monopolistic region (i.e., $n-k$ grows indefinitely as r declines) in order to avoid the future prospect of falling back, and the leader stops investing only when it expects to stay in the monopolistic region for a sufficiently long time. As $r \to 0$, a leader behaves as if it is infinitely patient. Even the distant threat of losing market power is perceived to be imminent; consequently, leaders scale back investment only if they expect to never leave the monopolistic region, causing market leadership to become endogenously permanent.

Why does a symmetric argument not apply to the follower? Consider the follower in state $k+1$. As $r \to 0$, $k+1$ grows indefinitely (Lemma 7), and the follower in this marginal state is further and further behind. Because both firms invest in all states 0 through k, the follower in state $k+1$ expects to fight a longer and longer war before it can reach state 0 and has a chance to become the leader. As the fight for leadership involves intense competition and large investment costs for a long time, the follower is eventually discouraged from the fight—when it is more than k steps behind—despite low r. Once again, the intense competition in states 0 through k dissipates future rents and serves as an endogenous deterrent to the follower in state $k+1$. Low interest rates motivate investment only if future leadership is attainable. As $r \to 0$ and as k grows, it becomes infinitely costly to overcome the competition in states 0 through k, and the prospect of becoming a future leader is perceived to be too low even for a patient follower in state $k+1$.

Theorem 1 is an aggregate result that builds on sharp analytical characterizations of the dynamic game between duopolists in each market. The duopolist game is rooted in models of dynamic patent races and is notoriously difficult to analyze: the state variable follows an endogenous stochastic process, and firms' value functions are recursively defined and therefore depend on flow payoffs and investment decisions in every state of the ergodic steady-state distribution $\{\mu_s\}_{s=0}^{n+1}$. Even seminal papers in the literature rely on numerical methods (e.g. Budd et al. (1993), Acemoglu and Akcigit (2012)) or restrictive simplifications to make the analysis tractable.[8] Relative to the literature, our analysis of an economy in a low-rate environment is further complicated by the fact that, as r declines, the ergodic state space $\{0, 1, \cdots, n+1\}$ becomes infinitely large.

In order to obtain Theorem 1, we fully characterize the asymptotic equilibrium as $r \to$

[8]For instance, Aghion et al. (2001) and Aghion et al. (2005) assume leaders do not invest in all $s \geq 1$, effectively restricting the ergodic state space as $\{0, 1\}$.

0. We analytically solve for the recursive value functions as a first-order approximation in r around $r = 0$, and we analytically characterize the rate at which equilibrium objects—value functions, investment cutoffs, the stationary distribution of productivity gaps—grow as $r \to 0$. Theorem 1 is a distillation of the full characterization, and we relegate the formal proof to the appendix. In what follows, we provide a sketch of the proof, in four steps. Each step aims to explain a specific feature in the shape of value functions shown in Figure 2. Note because total revenue in a market is always equal to 1, $rv_s \leq 1$ for all s.

Step 1: The leader's value in state $n+1$ is asymptotically large.

Formally, Lemma A.1 shows $\lim_{r\to 0} rv_{n+1} \to \pi_\infty - c\kappa > 0$. To see this, note the leader stops investing in state $n+1$ if and only if the marginal investment cost is higher than the change in value function, implying

$$c \geq v_{n+2} - v_{n+1} \geq \frac{\pi_{n+2} - rv_{n+1}}{r+\kappa}, \tag{13}$$

where the last inequality follows from rearranging the HJB equation (7) for state $n+2$. This in turn generates a lower bound for rv_{n+1}:

$$rv_{n+1} \geq \pi_{n+2} - c(r+\kappa) \xrightarrow{\text{(Lemma 7)}} \pi_\infty - c\kappa.$$

Step 2: The follower's value in state $k+1$ is asymptotically small.

Formally, Lemma A.2 shows $rv_{-(k+1)} \to 0$. To understand this, note the follower stops investing in state $k+1$ only if the marginal change in value function is lower than the investment cost ($v_{-k} - v_{-(k+1)} < c$). As $r \to 0$, the leader continues to invest in infinitely many states beyond k, and the follower stops investing in state $k+1$ despite knowing that once it gives up, the market structure tends to move in the leader's favor indefinitely, and that investing instead could delay or prevent falling back indefinitely. Lemma A.2 shows that follower not investing in state $k+1$ must imply follower's value in that state is asymptotically small.

Step 3: The value of a neck-and-neck firm is asymptotically small.

Formally, $rv_0 \to 0$. This is because as $k \to \infty$ (Lemma 7), firms in state zero expect to spend an indefinitely long time in the competitive region (states $s = 1, \ldots, k$), in which both firms invest at the upperbound, with a negative joint flow payoff due to intense competition. In fact, k must grow at a rate exactly consistent with an asymptotically small

v_0; this is because v_0 can be asymptotically large only if k grows slowly, but a large v_0 in turn implies that v_{-k} must be large—as the follower in state $-k$ is forward looking—which contradicts the earlier statement, that $rv_{-k} \to 0$. Conversely, the fact that v_0 must be non-negative—firms can always guarantee at least zero payoff—imposes an upper bound on the rate at which k diverges.

Step 4: A leader experiences an asymptotically large decline in value as it falls from the monopolistic region into the competitive region.

Formally, $\lim_{r\to 0} r(v_{k+1} - v_k) > 0$. This follows from the fact that v_{n+1} is asymptotically large (step 1) and v_0 is asymptotically small (step 3).

Step 4 implies that falling back into the competitive region is costly for the leader. Hence, starting from state $k+1$, the leader continues to invest in additional states in order to consolidate market power and reduce the future prospect of falling back. Its firm value increases as the productivity gap widens, and the leader stops only when the value function is sufficiently high, as characterized by inequalities (13). As a leader becomes infinitely patient, he must invest in sufficiently many states beyond k until the prospect of falling back into the competitive region vanishes, thereby endogenously perpetuating market leadership and causing the monopolistic region to become absorbing.

5 General Equilibrium and Quantitative Analysis

5.1 General Equilibrium

Up to this point, the analysis has taken the interest rate as exogenous, and we exogenously specify that the representative consumer has unit expenditure at each time t. We now embed the model into a general equilibrium framework by endowing the consumer with intertemporal preferences and endogenous income.

We limit our attention to the steady-state equilibrium, i.e., a balanced growth path, with aggregate productivity and consumption both growing at a constant rate g. Let $\hat{r}$ be the interest rate faced by the consumer. All of the formal statements in earlier sections continue to hold along the balanced growth path if we re-define $r \equiv \hat{r} - g$. In other words, what we have been calling "the interest rate" in earlier sections is the growth-adjusted interest rate in the context of general equilibrium, which, as we show, is also equal to the discount rate of the representative consumer.

Formally, the consumer has the following intertemporal preferences:

$$\max_{\{y_1(t;\nu),y_2(t;\nu),L(t)\}} \int_0^\infty e^{-\rho t}\left(\ln C\left(t\right) - L\left(t\right)\right) dt \tag{14}$$

$$\text{s.t. } C\left(t\right) = \exp\left(\int_0^1 \ln\left[y_1\left(t;\nu\right)^{\frac{\sigma-1}{\sigma}} + y_2\left(t;\nu\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} d\nu\right),$$

$$\int_0^1 p_1\left(t;\nu\right) y_1\left(t;\nu\right) + p_2\left(t;\nu\right) y_2\left(t;\nu\right) d\nu = w\left(t\right) L\left(t\right) + \Pi\left(t\right),$$

where ρ is the discount rate and $\Pi\left(t\right)$ is the total profit net of the investment cost accrued to producers in the economy.

We normalize the wage rate $w\left(t\right) \equiv 1$ for all t, and specify that production and the investment cost are both paid in labor. The labor market clearing condition is

$$L\left(t\right) = \int_0^1 \left[y_1\left(t;\nu\right)\lambda^{-z_1(t;\nu)} + y_2\left(t;\nu\right)\lambda^{-z_2(t;\nu)}\right] d\nu + \left(\sum_{s=1}^\infty \mu_s\left(t\right)\left(c\left(\eta_s\right) + c\left(\eta_{-s}\right)\right) + 2\mu_0\left(t\right) c\left(\eta_0\right)\right).$$

The consumption aggregator $C\left(t\right)$ in (14) is once again CES across varieties within each market and Cobb-Douglas across markets. Given our normalization, the consumer's intratemporal problem implies total expenditure on all consumption goods is equal to one, thereby inducing instantaneous demand functions that coincide with the preferences in (4) of Section 3. In addition, the intertemporal preferences in (14) imply an Euler equation:

$$g\left(t\right) \equiv \frac{d\ln C\left(t\right)}{dt} = \hat{r}\left(t\right) - \rho \tag{15}$$

where $\hat{r}\left(t\right)$ is the general equilibrium interest rate and $g\left(t\right)$ is the growth rate of aggregate consumption.

On a balanced growth path, the consumption price index $P\left(t\right)$ takes the same form as defined in Section 3.1, and it declines at a constant rate g relative to the numeraire; hence, the value function of a firm currently in state s is

$$\begin{aligned} v_s\left(t\right) &= \mathbb{E}\left[\int_0^\infty e^{-\hat{r}\tau}\left\{\frac{\pi\left(t+\tau\right) - c\left(t+\tau\right)}{P\left(t+\tau\right)/P\left(t\right)}\right\} d\tau \middle| s\right] \\ &= \mathbb{E}\left[\int_0^\infty e^{-(\hat{r}-g)\tau}\left\{\pi\left(t+\tau\right) - c\left(t+\tau\right)\right\} d\tau \middle| s\right] \end{aligned}$$

The model presented in earlier Sections 3 and 4 represents the production-side of this

economy, and the earlier analysis demonstrates an inverted-U relationship between g and $\hat{r} - g$. That production-side relationship and the consumer-side Euler equation (15) together pin down the interest rate $\hat{r}$ and aggregate productivity growth g on a balanced growth path.

The Euler equation implies that in any equilibrium, $\hat{r} - g$ must be equal to the consumer discount rate ρ. Consequently, the general equilibrium version of the main result of Theorem 1 states that, if the consumer discount rate ρ declines towards zero, then in the limit, $\hat{r} - g \to 0$, and aggregate productivity growth rate g must decline and converge to $\kappa \cdot \ln \lambda$ (which implies that $\hat{r} \to \kappa \cdot \ln \lambda$ as well). Aggregate investment must decline, along with market dynamism and the aggregate labor share; markets become more concentrated, with high levels of markups, profits, and firm inequality.

What could be the sources of a decline in ρ? We follow Krugman (1998) and note that a decline in ρ can be seen as a catch-all shock that stands in place for any secular changes on the consumer side that pushes consumers towards saving more and consuming less, including a change in preferences, tightened borrowing constraints (e.g., Eggertsson and Krugman (2012)), or structural shifts such as an aging population (e.g., Eggertsson et al. (2019)) and rising inequality (e.g., Summers (2014), Mian et al. (2020)). Hence, the model presents an alternative view of the reasons behind "secular stagnation." As in traditional secular stagnation explanations, an initial inward shift in the consumer-side curve can lower equilibrium interest rates to very low levels. However, "stagnation" is not due to monetary constraints such as the zero lower bound or nominal rigidities. Instead, a large fall in interest rates can make the economy more monopolistic for reasons laid out above, thereby lowering investment and productivity growth. This is depicted in Figure 4.

Figure 4: Growth and the interest rate in general equilibrium

5.2 Quantitative Analysis

This section explores the quantitative properties of the model, with two goals. The first goal is validation—the model is numerically solved with a convex investment cost function, and we show that the limiting properties of the steady-state in Theorem 1, as well as other qualitative features of the equilibrium discussed in Section 4, continue to hold as we dispense with the linear-investment-cost assumption. Second, we show that, despite its parsimony, the model has some quantitative bite in explaining long-run trends in productivity growth and the profit shares. The quantitative model also has the added benefit that it illustrates some of the main mechanisms of the model.

According to Theorem 1, the steady-state distance between leaders and followers diverges as $r \to 0$. Hence, under either Bertrand or Cournot competition, the steady-state profit share converges to one. For quantitative relevance, we continue to assume Bertrand competition but modify the microfoundation for flow profits $\{\pi_s\}$ as follows. We specify that the production cost of the follower is $\lambda^{\max\{s,\bar{s}\}}$ times the cost of the leader for some parameter $\bar{s}$; hence, while a greater distance always implies a bigger strategic advantage for the leader—it takes the follower more steps to catch up with the leader—a greater s only translates into an additional production cost advantage up to $s \leq \bar{s}$. For simplicity, we set $\bar{s} = 1$, so that flow profits for both firms are constant for all $s \geq 1$.

The calibration is purposefully simple with only four other parameters. The cost function is specified to be quadratic, $c(\eta_s) \equiv (c \cdot \eta_s)^2$, where c is a cost-shifter, and the investment space is assumed to be sufficiently large so that η_s is always interior. The other three parameters are κ, the rate of technological diffusion; λ, the step-size of productivity gains; σ, the elasticity of substitution between two firms in the same market. The parameters σ and λ jointly determine flow profits $\{\pi_s\}$, which, along with c and κ, determine the equilibrium growth rate.

The calibration is done using the general equilibrium version of the model, with $r \equiv \hat{r} - g$, i.e., a firm's discount rate r is indeed the real interest rate $\hat{r}$ minus the productivity growth rate g. The calibration of these parameters $\{c, \kappa, \sigma, \lambda\}$ targets four moments: TFP growth rate and profit shares in high- and low-interest rate steady-states. The high-interest rate steady-state represents the U.S. economy during the years 1984–2000, and the low-interest rate steady-state for the years 2001–2016. For TFP growth—1.10% in the high-$\hat{r}$ period and 0.76% in the low-$\hat{r}$ period—we use the unadjusted total factor productivity for the business sector from the Federal Reserve Bank of San Francisco's database (Fernald (2015)).

For the profit share, we target 0.14 in the high-$\hat{r}$ period and 0.17 in the low-$\hat{r}$ period,

and we compute it from our model as average profits net of investment cost relative to revenue across all firms. These profit shares translate into markups of 16% and 20%, respectively; they capture the rise in markups in the United States and correspond roughly to the midpoint of recent estimates in the literature.[9] Finally, for the real interest rate, the U.S. AA corporate bond rate net of current inflation is used, which is 4.69% for the high-$\hat{r}$ period and 1.09% for low-$\hat{r}$ period (Farhi and Gourio (2019)). We use the AA corporate bond rate instead of the 10-year treasury rate—3.94% and 1.06% in the two periods—because the former is more relevant as the firms' discount rate, but the quantification is not sensitive to this choice. Table 1 shows the parameter values for the model's fit.

Table 1: Calibration: Parameters and Model Fit

Definition	Parameter	Value	Moment	Target	Model
Elasticity of substitution	σ	12	TFP growth, high-$\hat{r}$	1.10%	1.09%
Productivity step size	λ	1.21	TFP growth, low-$\hat{r}$	0.76%	0.76%
Technology diffusion rate	κ	3.93	Profit share, high-$\hat{r}$	0.14	0.14
Investment cost shifter	c	33.4	Profit share, low-$\hat{r}$	0.17	0.17

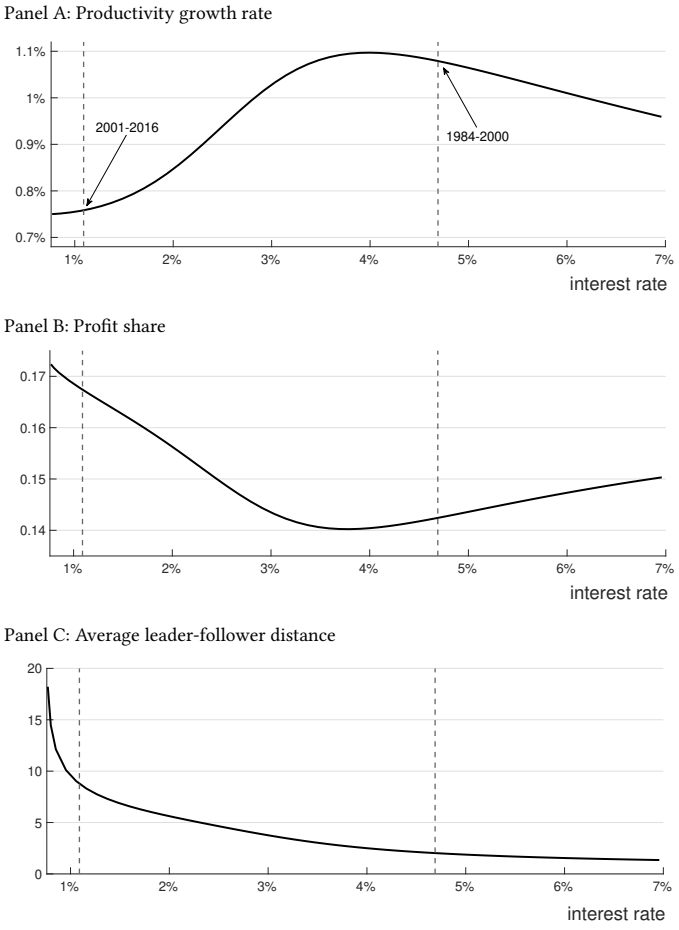
Figure 5 shows aggregate variables as steady states are compared for a decline in interest rates under the calibration. Panel A plots the productivity growth rate g against the interest rate $\hat{r}$.[10] There are three noteworthy features. First, as the theory predicts, g has an inverted-U relationship with $\hat{r}$. Moving from right to left, as $\hat{r}$ declines, g first increases as in traditional models. But eventually g declines. Second, in the limit as $\hat{r} - g \to 0$, g converges to $\kappa \cdot \ln \lambda$. This is a sharp prediction that is shown above analytically in Theorem 1 under the linear-investment-cost assumption. The numerical solution here shows that the prediction continues to hold under a convex investment cost. Note that this is not an artifact of the calibration, as we find g converges to $\kappa \ln \lambda$ under any calibration of the model. Third, growth is maximized at $g = 1.1\%$ when the real interest rate is around $\hat{r} = 4\%$. The productivity growth rate therefore starts to decline well above the limit, implying that the mechanism is empirically relevant.

Panel B of Figure 5 shows an U-shaped relationship between the net profit share and the interest rate. As $\hat{r}$ declines, competition always intensifies in any given state, but the leader-follower distance tends to widen. Intensified competition raises investment costs, whereas widening leader-follower-distance raises gross profits. As $\hat{r}$ declines (right to left

[9] See Gutiérrez and Philippon (2016, 2017); Hall (2018); Barkai (forthcoming); Edmond et al. (2019); De Loecker et al. (2020), among many others.

[10] Recall that in general equilibrium, $r \equiv \hat{r} - g$ is the discount rates used by firms when making investment decisions.

Figure 5: Comparative steady states: low interest rates on productivity growth, profit share, and average leader-follower distance

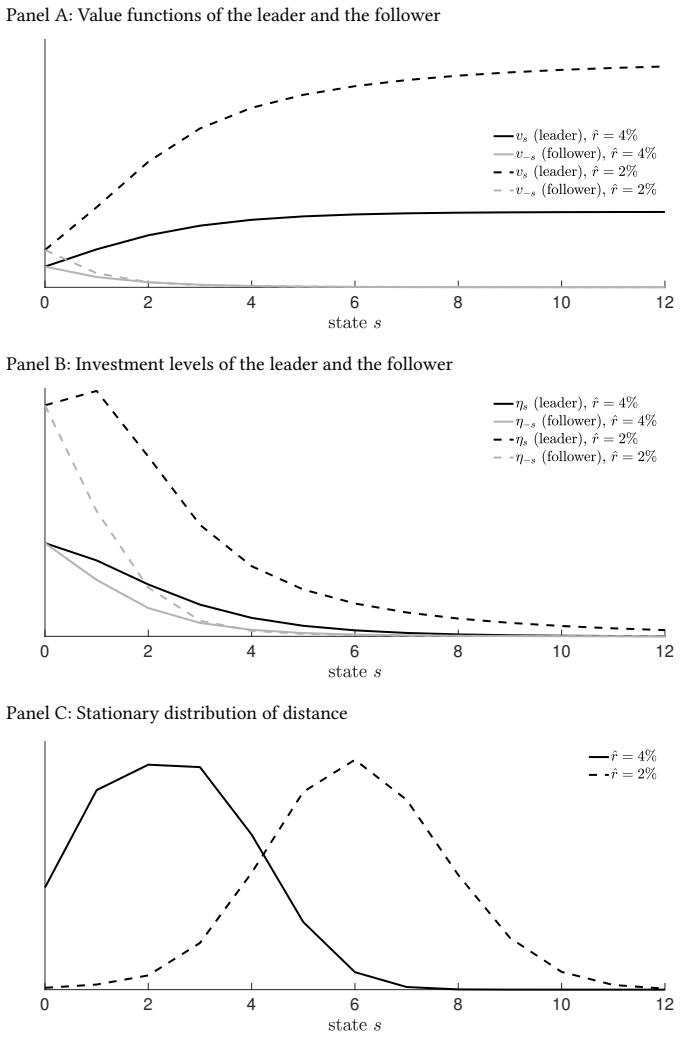


in the figure), initially the first force dominates and the profit share declines; eventually for $\hat{r}$ sufficiently low, the second force dominates, steady-state competition and investment decline, and the profit share increases.

Panel C of Figure 5 shows the average leader-follower distance monotonically increases as $\hat{r}$ declines and tends to infinity as $\hat{r} - g \to 0$. This is consistent with Theorem 1, which establishes the divergence in leader-follower distance analytically under a linear investment cost.

Figure 6 shows additional comparative steady state results by illustrating how state-by-state value functions (Panel A), investment levels (Panel B), and the stationary distribution of leader-follower distance (Panel C) vary at two levels of interest rates, $\hat{r} = 4\%$

Figure 6: Comparative steady states: state-by-state value functions, investment, and stationary distribution for $\hat{r} = 4\%$ (solid line) and $\hat{r} = 2\%$ (dashed line)



(solid line) and $\hat{r} = 2\%$ (dashed line). These figures confirm that the qualitative properties established analytically in Section 4 continue to hold under a convex investment cost. Panel A shows that the gain in the leader's value (black line) from state 0 to being far ahead (e.g. in state 12) is greater than the loss in the follower's value (grey line), and the asymmetry is greater under a lower interest rate. As explained above, this feature of the equilibrium value functions is due to the intensified competition between firms when their distance s is small, as shown in Panel B. Strategic competition therefore serves as

an endogenous motivator for the leader and a deterrent for the follower, resulting in the leader investing more than the follower in every state along the intensive margin. Panel B further demonstrates that the leader-follower investment gap widens in every state as r declines. Finally, Panel C plots the stationary distribution of firm distance $\{\mu_s\}$ and shows the distribution undergoes a first-order-stochastic-dominant shift to the right as r declines.

6 Additional Discussion

This section presents a number of extensions. The policy implications of the framework are discussed in Section 6.1. Section 6.2 discusses the implication of introducing real-world financial frictions, and Section 6.3 discusses the key "no-leapfrog" feature of the model and its relevance in the real world. Section 6.4 discusses transitional dynamics and the model's asset-pricing implications.

6.1 Policy Implications

The main result in Theorem 1, that $\lim_{r\to 0} g = \kappa \cdot \ln \lambda$, has interesting implications for antitrust policies in a low interest rate environment. As with traditional endogenous growth models, it is the incentive to gain market power that drives investment and growth in this framework. The additional insight in the model studied here is that investment by market leaders responds more aggressively to lower interest rates than the investment by market followers. Correspondingly, a low interest rate environment creates an expectation that market leaders will fight much more fiercely if market followers were to try to close in on the leader. This expectation of tougher competition, and the associated higher cost, discourages challengers from investing to unseat market leaders.

The expectation of tougher resistance by market leaders in a low interest rate environment reduces competition and growth. In these situations, regulation that reduces the expectation of tougher competition from market leaders can help raise investment and productivity growth. The model therefore shows why anti-trust regulation may become *more* important in a low interest rate environment.

But which anti-trust policies are most effective in this model? Broadly speaking, there are two types of potential policies that are relevant. The first type aims at helping market followers in terms of flow payoffs—such as taxing the leader's flow profits or subsidizing the follower's flow investment costs. The second type facilitates technological transfers from leaders to followers—such as policies that directly raise κ by restricting defensive

R&D and removing barriers for followers to compete.

In principle, policies focused directly on flow payoffs may promote investment by discouraging leaders' investment and encouraging followers'. However, as Theorem 1 suggests, these policies are ineffective at promoting investment and growth in a low interest rate environment. As $r \to 0$, the leader-follower strategic asymmetry continues to prevail, and the growth rate slows down to the same limit ($\kappa \cdot \ln \lambda$) whether these policies are in place or not. Because the strategic asymmetry is so strong, it is ineffective to merely *encourage* the followers; policies must target technological transfers directly by raising κ, thereby helping followers even as they become endogenously discouraged.

The calibrated model from section 5.2 can be used to demonstrate these intuitions. Two policies focused on flow payoffs are considered: one which taxes leader profits by 10% and the other which reduces the follower's investment cost by 10%. We also consider a policy that raises the rate of technological diffusion κ by 10%. Figure 7 shows the effects of these interventions; Panel A plots the relationship between the growth rate and the interest rate, and Panel B plots the relationship between the profit share and the interest rate. The baseline calibration is the solid line in black; the counterfactuals are represented in grey with various markers.

Figure 7: Counterfactual productivity growth and profit share: 10% tax on leader profits and 10% higher κ

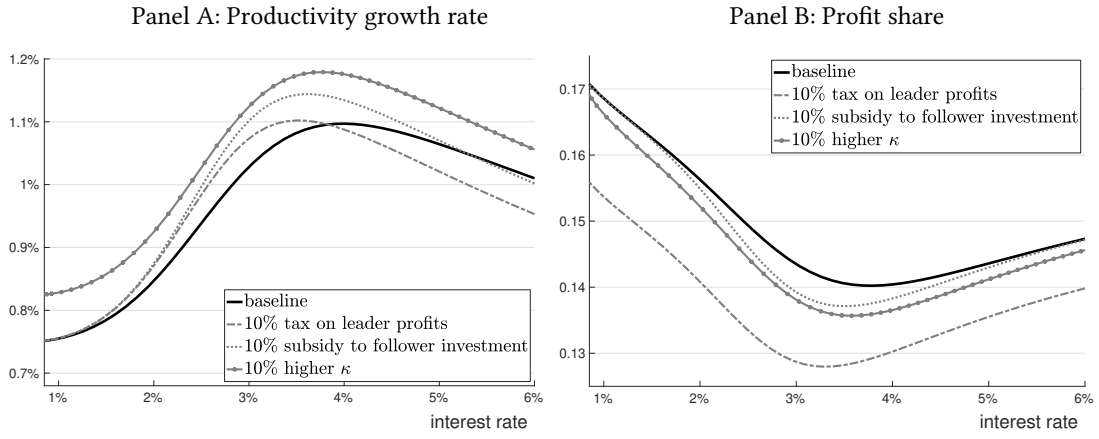


Figure 7 shows that taxing leader profits, while effective in reducing market power (Panel B), are not effective in stimulating investment and growth when the interest rate is sufficiently low (Panel A). Intuitively, the value to erecting barriers for a strategic advantage as r approaches zero is so large that even with 10% taxes, the value of being a permanent leader goes to infinity; market leaders therefore still have the incentive to com-

pletely discourage the followers. The intuition from Theorem 1 continues to hold under policies that constrain markups and profits, and, as $r \to 0$, growth will decline to the very same limit, $\kappa \cdot \ln \lambda$, regardless of whether the profit tax is in place or not. Taxing leader profits also has the undesirable effect that, for $\hat{r}$ sufficiently high (e.g. at the rates that prevail in 1984–2000) the policy also reduces growth by discouraging the incentive to become a leader. For similar reasons, subsidizing the follower's investment is also ineffective in promoting growth when the interest rate is low.

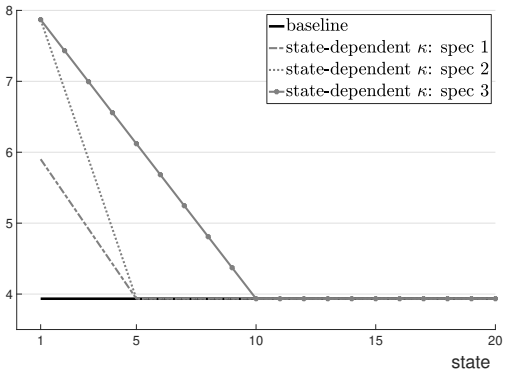
Antitrust policies focused on technological transfers on the other hand are effective in increasing investment and growth in our model. Theorem 1 implies that raising κ stimulates growth by directly raising the limiting growth rate of the economy. Figure 7 further shows that a higher κ raises the growth rate even when the interest rate is significantly above its lower limit. This is because a greater κ facilitates technological diffusion from leaders to followers, helping the followers even as they become endogenously discouraged. The steady-state therefore features more markets in states with stronger competition and greater investment, which leads to a higher aggregate growth rate. The fact that a higher κ implies more competitive markets is also evident in Panel B, which shows that the aggregate profit share is lower than the baseline for all levels of the interest rate, despite κ not directly affecting the flow profits in any given market.

Finally, we note that it is important for policy to raise κ in all states. If the rate of technology diffusion were state-dependent $\{\kappa_s\}_{s=1}^{\infty}$ and always finite—for instance, if policy facilitates technology transfer only if followers were not too far behind—then it is the limiting rate $\lim_{s\to\infty} \kappa_s$ that matters in for aggregate growth in a low interest rate environment: $\lim_{r\to 0} g = (\lim_{s\to\infty} \kappa_s) \cdot \ln \lambda$. Intuitively, because the leader-follower distance tends to diverge, bounded variations in κ_s for finite distance does not affect the steady-state as $r \to 0$.

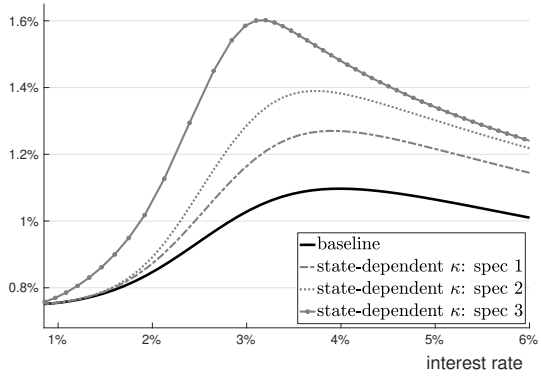
Figure 8 demonstrates this result. We consider three alternative policies that facilitate technology transfer but only for finite states. The state-dependent κ_s that these policies represent are shown in Panel A. Specification 1 sets κ_1 to be 50% higher than κ in the baseline calibration, and κ_s decays linearly towards the baseline over five states. Specification 2 sets κ_1 to be 100% higher than the baseline and κ_s again decays linearly over five states. Specification 3 sets κ_1 to be 100% higher than baseline and decays over 10 states. Panel B shows how steady-state growth rate varies with the interest rate under these policies. Evident from the figure, all three policies raise productivity growth when $r > 0$; however, the effectiveness declines as $r \to 0$, and, in the limit, the growth rate always converges to $\kappa \cdot \ln \lambda$.

Figure 8: Counterfactual productivity growth and profit share: state-dependent κ

Panel A: κ_s by state



Panel B: Productivity growth v.s. the interest rate



6.2 Introducing Financial Frictions

The model assumes that firms face no financial frictions. In particular, both the market leader and the follower use the same interest rate r to discount cash flows and neither faces an external financing premium. Financial frictions are intentionally assumed away to highlight that even when firms are not handicapped by an asymmetric financing constraint, the strategic incentive of market leaders becomes stronger as $r \to 0$, and such an incentive causes market power to increase in low-rate environments. We conjecture that introducing financial frictions would strengthen the core results of the model. Related points are made in the literature such as Caballero et al. (2008) and Gopinath et al. (2017).

Empirical evidence on financial frictions further suggests that financial frictions hurt market followers more than market leaders, especially in a low interest rate environment. In particular, a declining long-term interest rate is asssociated with a larger financing gap between industry leaders and followers. This fact is shown by constructing the interest rate faced by industry leaders (the top 5% of firms in any industry) versus industry followers in Compustat data. A firm's interest rate is calculated by dividing annual interest expense by total debt. Then the median imputed interest rate for industry leaders and followers is plotted in the left panel of Figure 9 over time, along with bootstrapped standard errors.

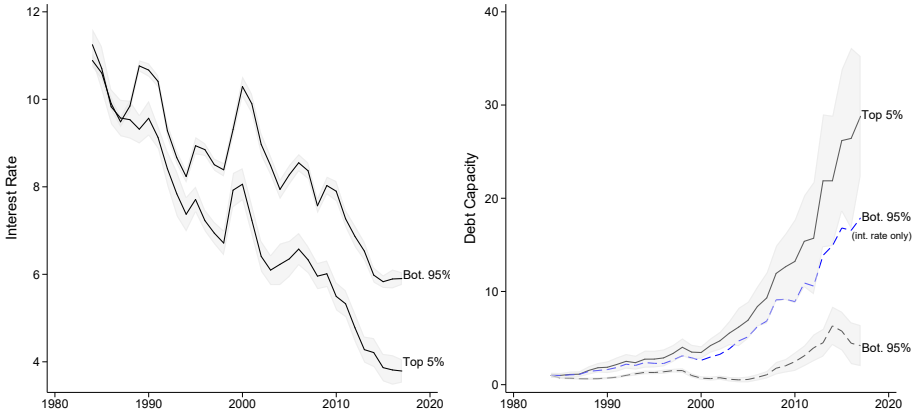
The figure shows that as the risk-free rate falls over time, the interest rate paid by the industry leaders and followers also falls. However, the spread in basis points increases as r declines. This shows that financing costs fall less than one-for-one for industry followers relative to industry leaders. The decline in the relative cost of borrowing for industry lead-

ers when the interest rate is low gives an additional reason to build a strategic advantage over followers.

This financing advantage that leaders enjoy can further be seen in the right panel of Figure 9. This figure plots the relative debt capacities for the median industry leader and follower in the Compustat data. Let π be EBIT for a firm, i the firm's interest rate, and D its maximum debt capacity. The maximum debt capacity D can be calculated using a minimum interest coverage ratio, k_{min} that lenders require, along with the formula $k_{min} = \frac{\pi}{i*D}$. A recent note from the Federal Reserve suggests that $k_{min} = 2$ (Palomino et al. (2019)).

The right panel of Figure 9 plots imputed debt capacity for the median industry leader and follower using the firm's interest rate and EBIT. Debt capacities are normalized to one at the beginning of sample. We can see that over time, as the long-term interest rate has fallen, the debt-capacity gap between industry leaders and followers has expanded considerably. The middle blue line shows how much of the gap is coming from the median leader and follower facing different interest rates alone. That is, it plots the follower's debt capacity assuming the follower continues to earn the leader's EBIT throughout the sample period. Figure 9 makes it clear that low interest rates have given industry leaders a large financing advantage over followers. They can use this advantage to, for example, threaten potential entrants with price wars or predatory acquisitions. All of this is assumed away in the model, but would likely strengthen the results if considered explicitly.

Figure 9: Interest rate and debt capacity for industry leaders and followers



6.3 Discussion of Model Assumptions

The key feature of the model that delivers the main result is that technological progress is incremental and follows a step-by-step process. In other words, the follower cannot "leapfrog" the leader in a single step. As explained earlier, it is the expectation of tougher competition for the follower when rates are low that discourages the follower relative to the leader as interest rates fall. For this expectation to remain relevant, investment today should bring the follower closer to leader, but it cannot allow the follower to leapfrog the leader regardless of how far back the follower is.

The condition of incremental innovation is plausible and relevant in a wide variety of contexts. Most of the innovation that happens is gradual and incremental, with each patent or scientific paper making an incremental contribution without creating a whole new paradigm. Recent empirical work by Bloom et al. (2020) suggests that if anything, innovation may be becoming more incremental and gradual in recent years. Moreover, low interest rates in a leapfrogging world would raise investment levels which is counterfactual.

The "no leapfrogging" condition is also realistic in that it helps to understand the real-world phenomena of market leaders conducting defensive R&D, erecting entry barriers, and engaging in predatory acquisitions. In the model, market leaders invest not only for higher flow profits but, importantly, also to acquire a strategic advantage and to prolong leadership—the main theorem holds even if a leader's flow profit does not increase with distance, e.g. when $\pi_s = \pi_\infty > 0 \; \forall s \geq 1$. The model's insight also helps to explain the ever expanding "kill zone" around industry giants' area of influence that makes it difficult for young startups to thrive (Cunningham et al. (2019)). As the Economist headlined in its report on June 2, 2018, "American tech giants are making life tough for startups". One can show that in our model, if market followers always leapfrog the leader with one successful investment, the leader no longer has the incentive to create such an empirically realistic strategic advantage. Instead, the leader invests only to acquire higher flow profits.

Nonetheless, the "no leapfrogging" condition cleanly identifies the scope and limit of the theoretical result. An economy can break-out of the low investment and low productivity equilibrium in a low interest rate environment if there appears on the horizon the possibility of investing in paradigm-shifting technology that will enable followers to leapfrog leaders (e.g., Cabral (2018)). However, if such paradigm-shifting opportunities are rare, or only apply to a small set of industries, the insight from the framework will continue to hold.

Finally, it is important to note that the key results are insensitive to other auxiliary fea-

tures of the model. Figure 7 shows that productivity growth converges to the same limit $\kappa \ln \lambda$ under a convex cost function, alternative profit levels, and follower cost advantage (i.e., state-dependent cost function). Figure 8 further shows that, when the rate of technology diffusion is state-dependent, growth converges to $\lim_{s\to\infty} \kappa_s \cdot \ln \lambda$. Intuitively, Theorem 1 characterizes the asymptotic equilibrium as $r \to 0$; consequently, bounded variations of κ_s in finite states do not affect firms' decisions in the limit.

6.4 Transitional Dynamics and Asset-Pricing Test

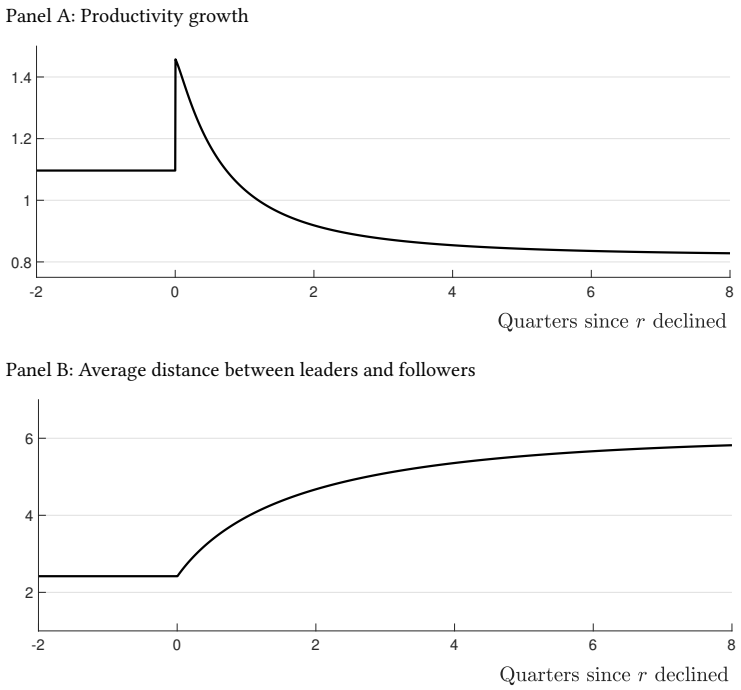
This study focuses mainly on the analysis of steady-states of the model. How long does it take for the economy to transition from one steady-state into another, following an unexpected and permanent interest rate shock? Figure 10 answers this question by showing the impulse response of a decline in the interest rate from 4% to 2%. Panel A shows the time path of productivity growth and Panel B is for the average productivity gap between leaders and followers. Starting from a steady-state, a permanent decline in the interest rate immediately moves market participants to a new equilibrium, featuring higher investments and productivity growth given any productivity gap (Panel A). The average productivity gap starts to rise, although it moves slowly (Panel B). Over time, as the distribution of the state variable converges to the new steady-state and as the average productivity gap increases, the equilibrium growth rate and investment eventually decline to the new steady-state level.

Figure 10 shows the convergence is rapid. Productivity growth is 1.1% in the initial steady-state and 0.82% in the new-steady-state; Panel A shows that it takes about 1.5 quarters for the growth rate to decline to 0.96%, closing about half of the steady-state difference. The initial boost in productivity growth lasts only 0.75 quarters, after which the growth rate declines below 1.1%.

A companion paper (Liu et al. (2020)) examines the transitional dynamics of the model for an unexpected shock to the interest rate. It shows that, starting from a steady-state with a low interest rate, an unexpected but permanent decline in the interest rate benefits industry leaders more than industry followers, and this asymmetric effect becomes stronger at lower levels of the initial interest rate. In the language of asset pricing, the model predicts that, when interest rates are low, market leaders have higher "duration"—log-sensitivity of firm valuation to the interest rate—and also higher "convexity"—the second derivative of log-valuation with respect to the interest rate.

The companion study tests this hypothesis using CRSP-Compustat merged data from 1962 onward. It constructs a "leader portfolio" that is long industry leaders and is short

Figure 10: Impulse response: reduction of interest rate from 4% to 2%



industry followers, and it examines the portfolio's performance in response to quarterly changes in interest rates. As the analysis there shows, the leader portfolio exhibits higher returns in response to a decline in interest rates for interest rates below a threshold, and this response becomes stronger at lower levels of the initial interest rate. Therefore, the model's asset pricing implications of a decline in the interest rate are supported in the data.

7 Conclusion

This study highlights a new strategic force for the determination of firm investment in productivity enhancement. This strategic force leads to an asymmetric investment response of market leaders to market followers when interest rates fall to low levels. Market leaders aggressively invest to escape competition when interest rates are low, whereas market followers become discouraged by the fierce competition that would be necessary to gain market leadership.

This strategic force delivers a unified explanation for the presence across advanced

economies of low interest rates, high market concentration, high profits, large productivity gaps between market leaders and followers, and low productivity growth. The slowdown in productivity growth has been pervasive across almost all advanced economies. The slowdown started well before the Great Recession, suggesting that cyclical forces related to the crisis are unlikely to be the trigger. Furthermore, the slowdown in productivity is highly persistent, lasting well over a decade. The long-run pattern suggests that explanations relying on price stickiness or the zero lower bound on nominal interest rates are less likely to be the complete explanation. This paper introduces the possibility of low interest rates as the common global factor that can potentially explain the slowdown in productivity growth.

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A Appendix: Proofs

A.1 Proof of claims in Sections 2 and 3

Proof of Proposition 1 The solution to HJB equations (1) through (3) imply that equilibrium investment and value functions must satisfy $\eta_s = v_{s+1} - v_s$ for $s \in \{-1, 0, 1\}$. The HJB equations can thus be re-written as

$$(r + \eta_s/2 + \eta_{-s})\, v_s = \pi_s + \eta_s v_{s+1}/2 + \eta_{-s} v_{s-1} \qquad \text{for } s \in \{-1, 0, 1\}\,. \tag{A.1}$$

Substitute using $v_2 = \pi_2/r$, $v_{-2} = \pi_{-2}/r$, $v_1 = v_2 - \eta_1$, $v_0 = v_2 - \eta_1 - \eta_0$, and $v_{-1} = v_2 - \eta_1 - \eta_0 - \eta_{-1}$, the HJB equations become a system of three quadratic equations involving three endogenous variables $\{\eta_{-1}, \eta_0, \eta_1\}$ with exogenous parameters $\{\pi_s\}$ and r. That $d\eta_s/dr < 0$ follows from totally differentiating the system of equations and applying the implicit function theorem.

We prove a generalized version of the limiting result that as $r \to 0$, $\eta_1 \to \infty$, $\eta_{-1} \to \infty$, and $(\eta_1 - \eta_{-1}) \to \infty$, under a quadratic cost function with a leader disadvantage. Specifically, we define $c_s = 1$ if $s < 1$ and $c_s = c$ if $s = 1$, and we write the HJB equation for state $s \in \{-1, 0, 1\}$ as

$$r v_s = \max_{\eta} \pi_s - c_s \eta^2/2 + \eta\,(v_{s+1} - v_s) + \eta_{-s}\,(v_{s-1} - v_s)\,.$$

The parameter c is a cost shifter for the leader. The example in Section 2 has $c = 1$. When $c > 1$, leader holds a cost disadvantage relative to the follower. We now prove the limiting result for a generic c. Note optimal investment satisfies $\eta_{-1} = v_0 - v_{-1}$, $\eta_0 = v_1 - v_0$, and $c\eta_1 = v_2 - v_1$. After substituting these expressions into the HJB equation and then taking the limit $r \to 0$, we obtain

$$v_1 \sim \frac{\eta_1 v_2 + 2\eta_{-1} v_0}{\eta_1 + 2\eta_{-1}}, \qquad v_0 \sim \frac{v_1 + 2v_{-1}}{3}, \qquad v_{-1} \sim \frac{\eta_{-1} v_0 + 2\eta_1 v_{-2}}{\eta_{-1} + 2\eta_1},$$

where we use $x \sim y$ to denote $\lim_{r \to 0} (x - y) = 0$. Using optimal investment decisions to substitute out v_{-1}, v_0 and v_1, we obtain

$$c\eta_1 \sim \frac{8\eta_{-1}\,(v_2 - v_{-2})}{6\eta_1 + 9\eta_{-1}}, \qquad \eta_{-1} \sim \frac{2\eta_1\,(v_2 - v_{-2})}{6\eta_1 + 9\eta_{-1}},$$

thereby implying $c\eta_1^2 \sim 4\eta_{-1}^2$. As $r \to 0$, $v_2 - v_{-2} \to \infty$, implying that $\eta_1 \to \infty$, $\eta_{-1} \to \infty$, and $(\eta_1 - \eta_{-1}) \to \infty$ if and only if $c < 4$. In particular, when the leader does not have a cost disadvantage ($c = 1$), the difference between leader and follower investment diverges.

Proof of Lemmas 1 and 2 The CES demand within each market implies that the market share of firm i is $\delta_i \equiv \frac{p_i y_i}{p_1 y_1 + p_2 y_2} = \frac{p_i^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}$. Under Bertrand competition, the price of a firm with productivity z_i must solve $p_i = \frac{\sigma(1-\delta_i)+\delta_i}{(\sigma-1)(1-\delta_i)}\lambda^{-z_i}$, with markup $m_i \equiv \frac{p_i}{\lambda^{-z_i}} = \frac{\sigma(1-\delta_i)+\delta_i}{(\sigma-1)(1-\delta_i)}$ and profits $\pi_i = \delta_i\left(\frac{p_i - \lambda^{-z_i}}{p_i}\right)$. Now define ρ_s as the relative price between leader and follower in a market with productivity gap s. Taking ratios of the prices and re-arrange, we derive that ρ_s must solve $\rho_s^\sigma = \lambda^{-s}\frac{\left(\sigma\rho_s^{\sigma-1}+1\right)}{\sigma+\rho_s^{\sigma-1}}$. Market share is therefore $\delta_s = \frac{\rho_s^{1-\sigma}}{\rho_s^{1-\sigma}+1}$ for the leader and $\delta_{-s} = \frac{1}{\rho_s^{1-\sigma}+1}$ for the follower and profits are $\pi_s = \frac{1}{\sigma\rho_s^{\sigma-1}+1}$ and $\pi_{-s} = \frac{\rho_s^{\sigma-1}}{\sigma+\rho_s^{\sigma-1}}$, respectively. Leader's markup is $m_s = \frac{\sigma+\rho_s^{1-\sigma}}{\sigma-1}$ and follower's markup is $m_{-s} = \frac{\sigma\rho_s^{1-\sigma}+1}{(\sigma-1)\rho_s^{1-\sigma}}$.

The fact that follower's flow profits are convex in s follows from algebra. Moreover, $\lim_{s\to\infty}\rho_s^\sigma\lambda^s = 1/\sigma$; hence, for large s, $\pi_s \approx \frac{1}{\sigma^{\frac{1}{\sigma}}\lambda^{-\frac{\sigma-1}{\sigma}s}+1}$ and $\pi_{-s} \approx \frac{1}{\sigma^{\frac{2\sigma-1}{\sigma}}\lambda^{\frac{\sigma-1}{\sigma}s}+1}$. The eventual concavity of π_s and $(\pi_s + \pi_{-s})$ as $s \to \infty$ is immediate. Also note that, as $s \to \infty$, $\pi_s \to 1$, $\pi_{-s} \to 0$, $m_s \to \infty$, $m_{-s} \to 0$.

Proof of Lemma 3 The expression $g = \ln\lambda\left(\sum_{s=0}^{\infty}\mu_s\eta_s + \mu_0\eta_0\right)$ shows that aggregate growth is equal to $\ln\lambda$ times the weighted-average investment rate of firms at the frontier—leaders and neck-and-neck firms. In a steady-state, the growth rate of the productivity frontier must be the same as the growth rate of followers; hence, aggregate growth rate g can also be written as $g = \ln\lambda\left(\sum_{s=1}^{\infty}\mu_s\left(\eta_{-s}+\kappa\right)\right)$.

To prove the expression formally, we proceed in two steps. First, we express aggregate productivity growth as a weighted average of productivity growth in each market. We then use the fact that, given homothetic within-market demand, if a follower in state s improves productivity by one step (i.e. by a factor λ) and a leader in state $s-1$ improves also by one step, the net effect is equivalent to one step improvement in the overall productivity of a single market.

Let $p(\nu) \equiv \left[p_1(\nu)^{1-\sigma} + p_2(\nu)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ be the price index of a single market ν. We can equivalently index for markets not using ν but instead using $\left(s, z^F\right)$, the productivity gap and the productivity of the follower. The growth rate g of aggregate productivity defined

in (12) is equal to $-\frac{d\ln P}{dt}$, where P is the ideal consumer price index, and can be written as:

$$g \equiv \frac{d\ln\lambda^Z}{dt} = -\frac{d\ln P}{dt} = -\frac{d\int_0^1 \ln p\left(\nu\right)d\nu}{dt} = -\sum_{s=0}^{\infty}\mu_s \times \frac{d\left[\int_{z^F}\ln p\left(s,z^F\right)dF\left(z^F\right)\right]}{dt}.$$

Now recognize that productivity growth rate in each market, $-\frac{d\ln p\left(s,z^F\right)}{d\ln t}$, is a function of only the productivity gap s and is invariant to the productivity of follower, z^F. Specifically, suppose the follower in market $\left(s,z^F\right)$ experiences an innovation, the market price index becomes $p\left(s-1,z^F+1\right)$. If instead the leader experiences an innovation, the price index becomes $p\left(s+1,z^F\right)$. The corresponding log-changes in price indices are respectively

$$\begin{aligned} a_s^F &\equiv \ln p\left(s-1,z^F+1\right)-\ln p\left(s,z^F\right) = -\ln\lambda+\ln\left[\rho_{s-1}^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}-\ln\left[\rho_s^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}, \\ a_s^L &\equiv \ln p\left(s+1,z^F\right)-\ln p\left(s,z^F\right) = \ln\left[\rho_{s+1}^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}-\ln\left[\rho_s^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

where ρ_s is the implicit function defined in the proof for Lemma 1. The log-change in price index is independent of z^F in either case. Hence, over time interval $[t,t+\Delta]$, the change in price index for markets with state variable s at time t follows

$$\Delta\ln p\left(s,z^F\right) = \begin{cases} a_s^L & \text{with probability } \eta_s\Delta, \\ a_s^F & \text{with probability } \left(\eta_{-s}+\kappa\cdot\mathbf{1}\left(s\neq 0\right)\right)\Delta. \end{cases}$$

The aggregate productivity growth can therefore be written as

$$g = -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty}\mu_s\times\left(\eta_s a_s^L+\left(\eta_{-s}+\kappa\right)a_s^F\right),$$

where $a_0 \equiv a_0^F = a_0^L$. Finally, note that if both leader and follower in a market experiences productivity improvements, regardless of the order in which these events happen, the price index in the market changes by a factor of λ^{-1}: $a_s^F + a_{s-1}^L = a_s^L + a_{s+1}^F = -\ln\lambda$ for

all $s \geq 1$. Hence,

$$\begin{aligned} g &= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + (\eta_{-s} + \kappa)\, a_s^F\right) \\ &= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + (\eta_{-s} + \kappa)\left(-\ln\lambda - a_{s-1}^L\right)\right) \\ &= \ln\lambda \cdot \sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa\right) - \left(\sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L - a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) + \mu_0 2\eta_0 a_0\right). \end{aligned}$$

Given that steady-state distribution $\{\mu_s\}$ must follow equations (10) and (11), we know

$$\sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L - a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) + \mu_0 2\eta_0 a_0 = \sum_{s=1}^{\infty} \mu_s \eta_s a_s^L + \mu_0 2\eta_0 a_0 - \left(\sum_{s=1}^{\infty} \mu_s a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) = 0.$$

Hence aggregate growth rate simplifies to $g = \ln\lambda \cdot \sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa\right)$, which traces the growth rate of productivity laggards. We can also apply equations (10) and (11) again to express productivity growth as a weighted average of frontier growth: $g = \ln\lambda \cdot \left(\sum_{s=1}^{\infty} \mu_s \eta_s + 2\mu_0\eta_0\right)$.

A.2 Proof of claims in Sections 4.2 and 4.3

Section 4 maintains the assumption that investment cost is linear, $c\left(\eta_s\right) = c \cdot \eta_s$ for $\eta_s \in [0, \eta]$. As discussed in Section 3.2, we assume the investment space is sufficiently large—$c\eta > \pi_\infty$ and $\eta > \kappa$—so that firms can compete intensely if they choose to—and c is not prohibitively high relative to the gains from becoming a leader ($c\kappa < \pi_\infty - \pi_0$)—otherwise no firm has any incentive to ever invest.

Proof of Lemma 4 Recall $n+1$ is the first state in which market leaders choose not to invest, and $k+1$ is the first state in which followers choose not to invest: $n+1 \equiv \min\{s | s \geq 0, \eta_s < \eta\}$ and $k+1 \equiv \min\{s | s \leq 0, \eta_s < \eta\}$. Suppose $n < k$, i.e. leader invests in states 1 through n whereas follower invests in states 1 through at least $n+1$. We first show that, if these investment decisions were optimal, the value functions of both leader and follower in state $n+1$ must be supported by certain lower bounds. We then reach for a contradiction, showing that, if $n < k$, then market power is too transient to support these lower bounds on value functions.

The HJB equation for the leader in state $n+2$ implies

$$\begin{aligned} rv_{n+2} &= \max_{\eta_{n+2}\in[0,\eta]} \pi_{n+2} + \eta_{n+2}\left(v_{n+3} - v_{n+2} - c\right) + \left(\eta_{-(n+2)} + \kappa\right)\left(v_{n+1} - v_{n+2}\right) \\ &\geq \pi_{n+2} + \left(\eta + \kappa\right)\left(v_{n+1} - v_{n+2}\right). \end{aligned} \tag{A.2}$$

That the leader does not invest in state $n+1$ implies $c \geq v_{n+2} - v_{n+1}$; combining with (A.2) to obtain

$$rv_{n+1} \geq \pi_{n+2} - c\left(\eta + \kappa + r\right).$$

The HJB equation for the follower in state $n+1$ implies

$$\begin{aligned} rv_{-(n+1)} &= \max_{\eta_{-(n+1)}\in[0,\eta]} \pi_{-(n+1)} + \left(\eta_{-(n+1)} + \kappa\right)\left(v_{-n} - v_{-(n+1)}\right) - c\eta_{-(n+1)} \\ &\geq \pi_{-(n+1)} + \kappa\left(v_{-n} - v_{-(n+1)}\right). \end{aligned} \tag{A.3}$$

That the follower invests in state $n+1$ implies $c \leq v_{-n} - v_{-(n+1)}$; combining with (A.3) to obtain

$$rv_{-(n+1)} \geq \pi_{-(n+1)} + c\kappa. \tag{A.4}$$

Combining this with the earlier inequality involving rv_{n+1}, we obtain an inequality on the joint value $w_{n+1} \equiv v_{n+1} + v_{-(n+1)}$:

$$rw_{n+1} \geq \pi_{n+2} + \pi_{-(n+1)} - c\left(\eta + r\right) \tag{A.5}$$

We now show that inequalities (A.4) and (A.5) cannot both be true. To do so, we construct alternative economic environments with value functions $\hat{w}_1^{(0)}$ and $\hat{v}_{-1}^{(0)}$ such that $\hat{w}_1^{(0)} \geq w_{n+1}$ and $\hat{v}_{-1}^{(0)} \geq v_{-(n+1)}$; we then show that even these dominating value functions $\hat{w}_1^{(0)}$ and $\hat{v}_{-1}^{(0)}$ cannot satisfy both inequalities.

First, fix n and fix investment strategies (leader invests until state $n+1$ and follower invests at least through $n+1$); suppose for all states $1 \leq s \leq n+1$, follower's profits are equal to $\pi_{-(n+1)}$ and leader's profits are equal to π_{n+2}; two firms each earn $\frac{\pi_{-(n+1)}+\pi_{n+2}}{2}$ in state zero. The joint profits in this modified economic environment are independent of the state by construction; moreover, the joint flow profits always weakly dominate those in the original environment and strictly dominate in state zero ($\pi_{n+2} + \pi_{-(n+1)} \geq \pi_1 + \pi_{-1} > 2\pi_0$). Let $\hat{w}_s$ denote the value function in the modified environment; $\hat{w}_s > w_s$ for all $s \leq n+1$.

Consider the joint value in this modified environment but under alternative investment strategies. Let $\bar{n}$ index for investment strategies: leader invests in states 1 through $\bar{n}$ whereas the follower invests at least through $\bar{n}+1$. Let $\hat{w}_s^{(\bar{n})}$ denote the joint value in state s under investments indexed by $\bar{n}$. We argue that $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$. To see this, note the joint flow payoffs in all states 0 through $\bar{n}$ is constant by construction and is equal to $x \equiv \left(\pi_{n+2}+\pi_{-(n+1)}-2c\eta\right)$—total profits net of investment costs—and the joint flow payoff in state $\bar{n}+1$ is $\left(\pi_{n+2}+\pi_{-(n+1)}-c\eta\right) = x+c\eta$. $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is equal to a weighted average of x/r and $(x+c\eta)/r$, and the weight on $(x+c\eta)/r$ is higher when $\bar{n}$ is smaller. Hence, $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$, and that $\hat{w}_1^{(0)} \geq \hat{w}_{n+1}^{(n)} > w_{n+1}$. The same logic also implies $\hat{v}_0^{(0)} = \frac{1}{2}\hat{w}_0^{(0)} > \frac{1}{2}w_0 = v_0$.

Consider follower's value $\hat{v}_{-1}^{(0)}$ in the alternative environment, when investment strategies are indexed by zero, i.e. firms invest in states 0 and -1 only. We know $\hat{v}_{-1}^{(0)}$ must be higher than $v_{-(n+1)}$ because

$$\hat{v}_{-1}^{(0)} = \frac{\pi_{-(n+1)} - c\eta + \kappa\hat{v}_0^{(0)}}{r+\kappa+\eta} > \frac{\pi_{-(n+1)} - c\eta + \kappa v_0}{r+\kappa+\eta} \geq \frac{\pi_{-(n+1)} - c\eta + \kappa v_{-n}}{r+\kappa+\eta} = v_{-(n+1)}.$$

We now show that the inequalities $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ and $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c\left(\eta+r\right)$ cannot both hold. We can explicitly solve for the value functions from the HJB equations:

$$\hat{w}_0^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - 2c\eta + 2\eta\hat{w}_1^{(0)}}{r+2\eta}$$

$$\hat{w}_1^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - c\eta + \left(\eta+\kappa\right)\hat{w}_0^{(0)}}{r+\eta+\kappa}$$

$$\hat{v}_{-1}^{(0)} = \frac{\pi_{-(n+1)} - c\eta + \left(\eta+\kappa\right)\hat{w}_0^{(0)}/2}{r+\eta+\kappa}$$

Solving for $\hat{w}_1^{(0)}$ and $\hat{v}_{-1}^{(0)}$, we obtain

$$r\hat{w}_1^{(0)} = \pi_{n+2} + \pi_{-(n+1)} - c\eta\left(1 + \frac{\eta+\kappa}{r+3\eta+\kappa}\right)$$

$$\left(r+\eta+\kappa\right)r\hat{v}_{-1}^{(0)} = r\left(\pi_{-(n+1)} - c\eta\right) + \left(\eta+\kappa\right)\left(\frac{\pi_{n+2}+\pi_{-(n+1)}}{2} - c\eta\frac{r+2\eta+\kappa}{r+3\eta+\kappa}\right)$$

That $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ implies

$$(r+\eta+\kappa)\, r\hat{v}_{-1}^{(0)} = r\left(\pi_{-(n+1)} - c\eta\right) + (\eta+\kappa)\left(\frac{\pi_{n+2}+\pi_{-(n+1)}}{2} - c\eta\frac{r+2\eta+\kappa}{r+3\eta+\kappa}\right) \geq (r+\eta+\kappa)\left(\pi_{-(n+1)}\right.$$

$$\implies (\eta+\kappa)\left(\frac{\pi_{n+2}-\pi_{-(n+1)}}{2} - c\eta\frac{r+2\eta+\kappa}{r+3\eta+\kappa}\right) \geq (r+\eta+\kappa)\, c\kappa + c\eta r$$

Since $\frac{\pi_{n+2}-\pi_{-(n+1)}}{2} \leq \frac{\pi_{n+2}}{2} < c\eta$, it must be the case that

$$(\eta+\kappa)\, c\eta > (r+\eta+\kappa)\, c\kappa + c\eta r + (\eta+\kappa)\, c\eta\frac{r+2\eta+\kappa}{r+3\eta+\kappa}.$$

On the other hand, that $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c\,(\eta + r)$ implies $r \geq \eta\frac{\eta+\kappa}{r+3\eta+\kappa}$; hence the previous inequality implies

$$\begin{aligned}(\eta+\kappa)\, c\eta &> (r+\eta+\kappa)\, c\kappa + (\eta+\kappa)\, c\eta\frac{\eta}{r+3\eta+\kappa} + (\eta+\kappa)\, c\eta\frac{r+2\eta+\kappa}{r+3\eta+\kappa} \\ &= (r+\eta+\kappa)\, c\kappa + (\eta+\kappa)\, c\eta,\end{aligned}$$

which is impossible; hence $n \geq k$.

We now show that the follower does not invest in states $s \in \{k+1, ..., n+1\}$. First, note

$$\begin{aligned}(r+\eta+\kappa)\,(v_{-s} - v_{-s-1}) &= \pi_{-s} - \pi_{-s-1} + \kappa\,(v_{-s+1} - v_{-s}) + \eta\,(v_{-s-1} - v_{-s-2}) \\ &\quad + \max\left\{\eta\,(v_{-s+1} - v_{-s} - c)\,, 0\right\} - \max\left\{\eta\,(v_{-s} - v_{-s-1} - c)\,, 0\right\}.\end{aligned}$$

Suppose $v_{-s+1} - v_{-s} \geq (v_{-s} - v_{-s-1})$, then

$$(r+\eta+\kappa)\,(v_{-s} - v_{-s-1}) \geq \pi_{-s} - \pi_{-s-1} + \kappa\,(v_{-s+1} - v_{-s}) + \eta\,(v_{-s-1} - v_{-s-2})$$

$$\implies (r+\eta)\,(v_{-s} - v_{-s-1}) \geq \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2})\,.$$

If $v_{-s+1} - v_{-s} < (v_{-s} - v_{-s-1})$, then

$$\begin{aligned}(r+\eta)\,(v_{-s} - v_{-s-1}) &< \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2}) \\ &\quad + \max\left\{\eta\,(v_{-s+1} - v_{-s} - c)\,, 0\right\} - \max\left\{\eta\,(v_{-s} - v_{-s-1} - c)\,, 0\right\} \\ &\leq \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2})\,.\end{aligned}$$

To summarize, for all s,

$$v_{-s+1}-v_{-s} \geq (v_{-s}-v_{-s-1}) \iff (r+\eta)(v_{-s}-v_{-s-1}) \geq \pi_{-s}-\pi_{-s-1}+\eta(v_{-s-1}-v_{-s-2}) \tag{A.6}$$

Now suppose $\eta_{-k-1}=0$ but $\eta_{-s'}=\eta$ for some $s' \in \{k+2,...,n+1\}$. This implies

$$v_{-(k-1)}-v_{-k} \geq c > v_{-k}-v_{-k-1} < v_{-s'+1}-v_{-s'},$$

implying there must be at least one $s \in \{k+2,...,n+1\}$ such that $v_{-s+1}-v_{-s} \geq v_{-s}-v_{-s-1} < v_{-s-1}-v_{s-2}$. Applying (A.6),

$$(r+\eta)(v_{-s}-v_{-s-1}) \geq \pi_{-s}-\pi_{-s-1}+\eta(v_{-s-1}-v_{-s-2}) \tag{A.7}$$

$$(r+\eta)(v_{-s-1}-v_{-s-2}) < \pi_{-s-1}-\pi_{-s-2}+\eta(v_{-s-2}-v_{-s-3}) \tag{A.8}$$

Inequality (A.7) and $v_{-s}-v_{-s-1} < v_{-s-1}-v_{-s-2}$ implies $r(v_{-s}-v_{-s-1}) > \pi_{-s}-\pi_{-s-1}$; convexity in follower's profit functions further implies $r(v_{-s}-v_{-s-1}) > \pi_{-s-1}-\pi_{-s-2}$. Substitute into inequality (A.8), and using the fact $v_{-s}-v_{-s-1} < v_{-s-1}-v_{s-2}$, we deduce it must be the case that $(v_{-s-2}-v_{s-3}) > (v_{-s-1}-v_{-s-2})$. Applying (A.6) again,

$$(r+\eta)(v_{-s-2}-v_{-s-3}) < \pi_{-s-2}-\pi_{-s-3}+\eta(v_{-s-3}-v_{-s-4}).$$

That $r(v_{-s-2}-v_{-s-3}) > \pi_{-s-2}-\pi_{-s-3}$ further implies $(v_{-s-3}-v_{-s-4}) > (v_{-s-2}-v_{-s-3})$. By induction, we can show $v_{s-1}-v_{s-2} < v_{s-2}-v_{s-3} < \cdots < v_{-n}-v_{-(n+1)}$. But

$$(r+\eta+\kappa)\left(v_{-n}-v_{-(n+1)}\right) \leq \pi_{-n}-\pi_{-(n+1)}+\kappa(v_{-n+1}-v_{-n})+\eta(v_{-n+1}-v_{-n+1})$$

$$\implies (r+\eta)\left(v_{-n}-v_{-(n+1)}\right) \leq \pi_{-n}-\pi_{-(n+1)}$$

which is a contradiction, given convexity of the profit functions. Hence, we have shown $v_{-k}-v_{-(k+1)} \geq v_{-s}-v_{-s-1}$ for all $s \in \{k+1,...,n+1\}$, establishing that follower cannot invest in these states.

Proof of Lemma 5 Given the cutoffs (n, k), aggregate productivity growth is (from Lemma 3) $g = \ln \lambda \cdot \left(\sum_{s=1}^{n} \mu_s \eta + 2\mu_0 \eta\right)$. The steady-state distribution must follow

$$\mu_s \eta = \begin{cases} \mu_1 (\eta + \kappa)/2 & \text{if } s = 0 \\ \mu_{s+1} (\eta + \kappa) & \text{if } 1 \leq s \leq k-1 \\ \mu_{s+1} \kappa & \text{if } k \leq s \leq n+1 \\ 0 & \text{if } s > n+1 \end{cases} \tag{A.9}$$

Hence we can rewrite the aggregate growth rate as

$$\begin{aligned} g &= \ln \lambda \cdot \left(2\mu_0 \eta + \sum_{s=1}^{k-1} \mu_s \eta + \sum_{s=k-1}^{n} \mu_s \eta\right) \\ &= \ln \lambda \cdot \left(\mu_1 (\eta + \kappa) + \sum_{s=2}^{k} \mu_s (\eta + \kappa) + \sum_{s=k}^{n+1} \mu_s \kappa\right) \\ &= \ln \lambda \cdot \left(\mu^C (\eta + \kappa) + \mu^M \kappa\right), \end{aligned}$$

as desired. To solve for μ_0, μ^C, and μ^M as functions of n and k, we use (A.9) to write μ_s as a function of μ_{n+1} for all s. Let $\alpha \equiv \kappa/\eta$, then

$$\mu_s = \begin{cases} \mu_{n+1} \alpha^{n+1-s} & \text{if } n+1 \geq s \geq k \\ \mu_{n+1} \alpha^{n+1-k} (1+\alpha)^{k-s} & \text{if } k-1 \geq s \geq 1 \\ \mu_{n+1} \alpha^{n+1-k} (1+\alpha)^{k}/2 & \text{if } s = 0 \end{cases}$$

Hence $\mu_0 = \mu_{n+1} \alpha^{n+1-k} (1+\alpha)^k / 2$. The fraction of markets in the competitive and monopolistic regions can be written, respectively, as

$$\mu^M = \mu_{n+1} \sum_{s=k+1}^{n+1} \alpha^{n+1-s} = \mu_{n+1} \frac{1 - \alpha^{n-k+1}}{1-\alpha},$$

$$\mu^C = \mu_{n+1} \alpha^{n+1-k} \sum_{s=1}^{k} (1+\alpha)^{k-s} = \mu_{n+1} \alpha^{n-k} \left((1+\alpha)^k - 1\right).$$

Proof of Lemma 6 Given $k \geq 1$, the fraction of markets in the competitive region can be written as

$$\mu^C = \sum_{s=1}^{k} \mu_s = \mu_1 + \underbrace{\mu_1 (1+\alpha)^{-1}}_{=\mu_2} + \cdots + \underbrace{\mu_1 (1+\alpha)^{-(k-1)}}_{=\mu_k} = \underbrace{\mu_0 \frac{\kappa+\eta}{2\eta}}_{=\mu_1} \frac{1-(1+\alpha)^{-k}}{1-(1+\alpha)^{-1}} \geq \mu_0 \frac{\kappa+\eta}{2\eta}$$

Aggregate growth rate can be re-written as

$$g = \ln\lambda \cdot \left[(1-\mu_0)\kappa + \mu^C \eta\right] \geq \ln\lambda \cdot \left[(1-\mu_0)\kappa + \mu_0 \frac{\kappa+\eta}{2}\right] \geq \ln\lambda \cdot \kappa.$$

Aggregate investment is $I = 2\eta\left(\mu^C + \mu_0\right) + \eta\left(\mu^M - \mu_{n+1}\right)$. The definition of a steady-state implies $2\eta\mu_0 + \eta\left(\mu^M - \mu_{n+1}\right) = (\eta+\kappa)\mu^C + \kappa\mu^M$, thus $I = 2\eta\mu^C + \kappa(1-\mu_0) \geq \kappa$, as desired.

A.3 Proof of claims in Section 4.4

Consider the following recursive equations of value functions $\{u_s\}_{s=-\infty}^{\infty}$:

$$ru_{s+1} = \lambda_{s+1} + p_{s+1}(u_s - u_{s+1}) + q(u_{s+2} - u_{s+1}) \tag{A.10}$$

where λ_{s+1} is the flow payoff, p_{s+1} and q are respectively the Poisson rate of transition from state $s+1$ into state s and state $s+2$. Given u_s and $\Delta u_s \equiv u_{s+1} - u_s$, we can solve for all $u_{s+t}, t > 0$ as recursive functions of u_s and Δu_s. The recursive formulation generically does not have a closed-form representation. However, as $r \to 0$, the value functions do admit asymptotic closed form expressions, as Proposition A.1 shows. In what follows, let $\sim$ denote asymptotic equivalence as $r \to 0$, i.e. $x \sim y$ iff $\lim_{r\to 0}(x-y) = 0$.

Proposition A.1. Consider value functions $\{u_s\}_{s=-\infty}^{\infty}$ satisfying (A.10). Fix state s and integer $t > 0$.

Suppose $\lambda_{s'} \equiv \lambda$ and $p_{s'} \equiv p$ for all states $s \leq s' \leq t$. Let $\delta \equiv \frac{ru_s - \lambda}{q}$, $a \equiv \frac{p}{q}$, $b \equiv \frac{r}{q}$, then for all $t > 0$,

$$\begin{aligned} u_{s+t} - u_s \sim\ & (\Delta u_s)\frac{1-a^t}{1-a} + \delta\frac{t - \frac{a-a^t}{1-a}}{1-a} + \Delta u_s \cdot b\frac{(t-1)\left(1+a^t\right)(1-a) - (2-a)\left(a^t - a\right)}{(1-a)^3} \\ & + \delta b\frac{1}{(1-a)^3}\left(\frac{(t-2)(t-1)}{2}(1-a) - (t-3)a^t - a(2-a)(t-1) + 2a(1-a^t)\right) \end{aligned} \tag{A.11}$$

$$\begin{aligned} u_{s+t}-u_{s+t-1} \sim\ & \Delta u_s a^{t-1}+\delta\frac{1-a^{t-1}}{1-a}+\Delta u_s b\frac{\left((t-1)\left(1+a^{t}\right)-(t-2)\left(1+a^{t-1}\right)\right)}{(1-a)^2} \\ & -\Delta u_s b\frac{\left((2-a)\left(a^{t}-a^{t-1}\right)\right)}{(1-a)^3}+\frac{\delta b}{(1-a)^2}\frac{(t-2)(t-1)-(t-2)(t-3)}{2} \\ & -\frac{\delta b}{(1-a)^3}(t-3)a^{t}+(t-4)a^{t-1}-a(2-a)\Bigg). \end{aligned} \tag{A.12}$$

If $t\to\infty$ as $r\to 0$, then the formulas can be simplified as follows:

1. If $a<1$, then $u_{s+t}-u_{s+t-1}\sim\Delta u_s a^{t-1}+\frac{\delta}{1-a}+\frac{b\Delta u_s}{(1-a)^2}$; further,
   (a) if $r\Delta u_s\to 0$, then $u_{s+t}-u_s\sim\Delta u_s\frac{1}{1-a}+\frac{t\delta}{1-a}$;
   (b) if $r\Delta u_s\not\to 0$, then $r\left(u_{s+t}-u_s\right)\sim\frac{r\Delta u_s}{1-a}$.
2. Suppose $a>1$ and $r\Delta u_s\to 0$.
   (a) If $\Delta u_s+\frac{\delta}{a-1}\not\sim 0$, then $r\left(u_{s+t}-u_s\right)\sim\left(\Delta u_s+\frac{\delta}{a-1}\right)\frac{ra^t}{a-1}$ and $r\left(u_{s+t}-u_{s+t-1}\right)\sim\left(\Delta u_s+\frac{\delta}{a-1}\right)ra^{t-1}$.
   (b) If $\Delta u_s+\frac{\delta}{a-1}\sim 0$, then $u_{s+t}-u_s\sim-\frac{b\delta}{(1-a)^4}\cdot a^{t+1}$.

Suppose $\lambda_{s'}$ and $p_{s'}$ are state-dependent. Let $\lambda\geq\lambda_{s'}$ and $p\leq p_{s'}$ for all $s\leq s'\leq t$. The formulas in (A.11) and (A.12) provide asymptotic lower bounds for $u_{s+t}-u_{s+t-1}$ and $u_{s+t}-u_s$. Conversely, if $\lambda\leq\lambda_{s'}$ and $p\geq p_{s'}$ for all $s\leq s'\leq t$, then the formulas provide asymptotic upper bounds for $u_{s+t}-u_{s+t-1}$ and $u_{s+t}-u_s$.

Remark. Proposition A.1. expresses u_{s+t} and Δu_{s+t} as functions of u_s and Δu_s. One can also apply the Proposition write u_s and Δu_s as functions of Δu_{s+t} and u_{s+t}. Proposition A.1. thus enables us to solve for value functions asymptotically, and we apply it repeated throughout the rest of this appendix.

Proof of Proposition A.1. First suppose $\lambda_{s'}\equiv\lambda$ and $p_s\equiv p$ are constant for all states $s\leq s'\leq t$. Given u_s and Δu_s, we can solve for value functions u_{s+t} as

$$u_{s+1}-u_s=\Delta u_s$$

$$\begin{cases} u_{s+2}-u_{s+1}=a\Delta u_s+b\Delta u_s+\delta \\ u_{s+2}-u_s=(1+a)\Delta u_s+b\Delta u_s+\delta \end{cases} \tag{A.13}$$

$$\begin{cases} u_{s+3}-u_{s+2}=a^2\Delta u_s+(1+2a)\,b\Delta u_s+(1+a)\,\delta+o\left(r\right) \\ u_{s+3}-u_s=\left(1+a+a^2\right)\Delta u_s+(1+1+2a)\,b\Delta u_s+(1+1+a)\,\delta+b\delta+o\left(r\right) \end{cases}$$

where $o\left(r\right)$ captures terms that vanishes as $r\to 0$. Applying the formula iteratively, one can show that

$$u_{s+t+1}-u_{s+t}=a^t\Delta u_s+\delta\sum_{z=0}^{t-1}a^z+b\Delta u_s\sum_{z=1}^{t}za^{z-1}+b\delta\sum_{z=1}^{t-1}\sum_{m=1}^{z}ma^{m-1}+o\left(r\right)$$

$$u_{s+t+1}-u_s=\Delta u_s\sum_{z=0}^{t}a^z+\delta\sum_{z=0}^{t}\sum_{m=0}^{z-1}a^m+b\Delta u_s\sum_{z=1}^{t}\sum_{m=1}^{z}ma^{m-1}+b\delta\sum_{x=1}^{t-1}\sum_{z=1}^{x}\sum_{m=1}^{z}ma^{m-1}+o\left(r\right)$$

One obtains the proposition by applying the following formulas for power series summations:

1. $\sum_{z=0}^{t}a^z=\frac{1-a^{t+1}}{1-a}$;
2. $\sum_{z=0}^{t}\sum_{m=0}^{z-1}a^m=\frac{t+1-\frac{a-a^{t+1}}{1-a}}{1-a}$;
3. $\sum_{z=1}^{t}\sum_{m=1}^{z}ma^{m-1}=\frac{t\left(1+a^{t+1}\right)(1-a)-(2-a)\left(a^{t+1}-a\right)}{(1-a)^3}$;
4. $\sum_{x=1}^{t-1}\sum_{z=1}^{x}\sum_{m=1}^{z}ma^{m-1}=\frac{1}{(1-a)^3}\left(\frac{t(t-1)}{2}\left(1-a\right)-\left(t-2\right)a^{t+1}-a\left(2-a\right)t+2a\left(1-a\right)\right)$.

The third and fourth summations formulas follow because

$$\begin{aligned}\sum_{m=1}^{z}ma^{m-1} &= \left(1+2a+3a^2+\cdots+za^{z-1}\right)=\left(1-a^z+a\left(1-a^{z-1}\right)+\cdots+a^{z-1}\left(1-a\right)\right)/\left(1-a\right) \\ &= \left(1+a+\cdots+a^{z-1}-za^z\right)/\left(1-a\right)=\left(1-a^z-\left(1-a\right)za^z\right)/\left(1-a\right)^2\end{aligned}$$

$$\begin{aligned}\sum_{z=1}^{s}\sum_{m=1}^{z}ma^{m-1} &= \sum_{z=1}^{s}\left(1-a^z-\left(1-a\right)za^z\right)/\left(1-a\right)^2=\left(s-\left(a-a^{s+1}\right)-a\left(1-a\right)\sum_{z=1}^{s}za^{z-1}\right)/\left(1-a\right)^2 \\ &= \left(s-\left(a-a^{s+1}\right)-a\left(\left(1-a^s\right)/\left(1-a\right)-sa^s\right)\right)/\left(1-a\right)^2 \\ &= \left(s\left(1-a\right)-\left(a\left(1-a\right)-\left(1-a\right)a^{s+1}\right)-\left(a-a^{s+1}\right)+sa^{s+1}\left(1-a\right)\right)/\left(1-a\right)^3 \\ &= \left(s\left(1+a^{s+1}\right)\left(1-a\right)-\left(2-a\right)\left(a^{s+1}-a\right)\right)/\left(1-a\right)^3\end{aligned}$$

$$\begin{aligned}
&\sum_{x=1}^{s-1}\sum_{z=1}^{x}\sum_{m=1}^{z} ma^{m-1} = \sum_{x=1}^{s-1}\left(x\left(1+a^{x+1}\right)(1-a)-(2-a)\left(a^{x+1}-a\right)\right)/(1-a)^3 \\
= &\left(\sum_{x=1}^{s-1} x(1-a)+xa^{x+1}(1-a)-(2-a)\left(a^{x+1}-a\right)\right)/(1-a)^3 \\
= &\left(s(s-1)(1-a)/2+a^2(1-a)\sum_{x=1}^{s-1} xa^{x-1}-a(2-a)(s-1)-(2-a)a^2\frac{1-a^{s-1}}{1-a}\right)/(1-a)^3 \\
= &\left(\frac{s(s-1)}{2}(1-a)-(s-2)a^{s+1}-a(2-a)s+2a(1-a)\right)/(1-a)^3.
\end{aligned}$$

Now suppose λ_s and p_s are state-dependent, and $\lambda \geq \lambda_{s'}$, $p \leq p_{s'}$ for all $s \leq s' \leq t$. Let $\delta_s \equiv \frac{ru_s-\lambda_s}{q}$, $a_s \equiv \frac{p_s}{q}$ and note $\delta_s > \delta \equiv \frac{ru_s-\lambda}{q}$, $a_s > a$. By re-writing equations in this proof as inequalities (e.g. rewrite (A.13) as $u_{s+2}-u_{s+1} > a\Delta u_s + b\Delta u_s + \delta$ and $u_{s+2}-u_s > (1+a)\Delta u_s + b\Delta u_s + \delta$), the formulas in the Proposition provide asymptotic lower bounds for $u_{s+t}-u_{s+t-1}$ and $u_{s+t}-u_s$ as functions of u_s and Δu_s. Conversely, if $\lambda \leq \lambda_{s'}$ and $p \geq p_{s'}$ for all $s \leq s' \leq t$, then the formulas provide asymptotic upper bounds for $u_{s+t}-u_{s+t-1}$ and $u_{s+t}-u_s$. QED.

Proof of Lemma 7 Recall n and k are the last states in which the leader and the follower, respectively, chooses to invest in an equilibrium. Both n and k are functions of the interest rate r. Also recall that we use $w_s \equiv v_s + v_{-s}$ to denote the total firm value of a market in state s.

We first prove $\lim_{r\to 0} n = \infty$**.** Consider the sequence of value functions $\hat{v}_s$ generated by an alternative sequence investment decisions: leader follows equilibrium strategies and invests in n states whereas follower does not invest in any state. Under these alternative investments, flow payoff is higher in every state, hence the joint value of both firms is higher in every state—including state 0—thus $\hat{v}_0 \geq v_0$. One can further show by induction that the alternative value functions dominate the equilibrium value functions ($\hat{v}_s \geq v_s$) for all $s \geq 0$; intuitively, leader's value is higher in any state because it expects to spend more time in higher payoff states, since the follower does not invest. Also by induction one can show $\Delta v_s \geq \Delta \hat{v}_s$ for all $s \geq 0$; intuitively, when the follower does not invest, leader has less of an incentive to invest as well.

Now suppose n is bounded, and we look for a contradiction. Let N be the smallest integer such that (1) $N > n$ for all r, and (2) $\pi_N - \pi_0 > c\kappa$. Note $rv_N = r \cdot \frac{\pi_N + \kappa v_{N-1}}{r+\kappa} \to rv_{N-1}$ as $r \to 0$; hence $rv_N \sim rv_{N-1}$. By induction, because N is finite, $rv_s \sim rv_t \sim rv_{-s}$ for any $s, t \leq N$. Likewise, $r\hat{v}_s \sim r\hat{v}_t$ for any $s, t \leq N$. The fact that leader does not

invest in state $N-1$ implies $\lim_{r\to 0}(v_N - v_{N-1}) < c \Longrightarrow \lim_{r\to 0} rv_{N-1} > \pi_N - c\kappa$, which further implies $\lim_{r\to 0} r\hat{v}_0 \geq \lim_{r\to 0} rv_0 = \lim_{r\to 0} rv_{N-1} > \pi_N - c\kappa$. Also note that $\Delta\hat{v}_0 > \Delta\hat{w}_0 = \frac{r\hat{w}_1-(2\pi_0-2c\eta)}{r+2\eta} \to \frac{r\hat{w}_0-(2\pi_0-2c\eta)}{2\eta} = \frac{r\hat{v}_0-(\pi_0-c\eta)}{\eta}$. We now put these pieces together and apply Proposition A.1 to compute a lower bound for $\Delta\hat{v}_n$ as a function of $\hat{v}_0$ and $\Delta\hat{v}_0$ (substitute $u_s = \hat{v}_0$, $u_{s+t} = \hat{v}_N$, $a = \kappa/\eta$, $b = r/\eta$, $\delta = \frac{r\hat{v}_0-(\pi_N-c\eta)}{\eta}$):

$$\begin{aligned}
\lim_{r\to 0}\Delta\hat{v}_N &\geq \lim_{r\to 0}\left(\Delta\hat{v}_0\,(\kappa/\eta)^{N-1} + \frac{r\hat{v}_0-(\pi_N-c\eta)}{\eta}\frac{1-(\kappa/\eta)^{N-1}}{1-\kappa/\eta}\right)\\
&> \lim_{r\to 0}\frac{r\hat{v}_0-(\pi_0-c\eta)}{\eta}(\kappa/\eta)^{N-1} + \frac{r\hat{v}_0-(\pi_N-c\eta)}{\eta}\frac{1-(\kappa/\eta)^{N-1}}{1-\kappa/\eta}\\
&> \lim_{r\to 0}\frac{\pi_N-c\kappa-(\pi_0-c\eta)}{\eta}(\kappa/\eta)^{N-1} + \frac{\pi_N-c\kappa-(\pi_N-c\eta)}{\eta}\frac{1-(\kappa/\eta)^{N-1}}{1-\kappa/\eta}\\
&> \lim_{r\to 0} c\,(\kappa/\eta)^{N-1} + \frac{c\,(\eta-\kappa)}{\eta}\frac{1-(\kappa/\eta)^{N-1}}{1-\kappa/\eta}\\
&= c,
\end{aligned}$$

where the last inequality follows the fact that $\pi_N - \pi_0 > c\kappa$. Thus $\lim_{r\to 0}\Delta v_N \geq \lim_{r\to 0}\Delta\hat{v}_N > c$ and the leader must invest in state N, a contradiction.

Next, suppose $\lim_{r\to 0} k = \infty$ but $(n-k)$ remain bounded. Let $\epsilon \equiv 2c\eta - \pi_\infty > 0$. The joint flow payoff $\pi_s + \pi_{-s} - 2c\eta$ is negative and bounded above by $-\epsilon$ in all states $s \leq k$. As $k \to \infty$, if $n-k$ remain bounded, then there are arbitrarily many states in which the total flow payoffs for both firms is negative and only finitely many states in which the flow payoffs may be positive. The firm value in state 0 is therefore negative. Since firms can always ensure non-negative payoffs by not taking any investment, this cannot be an equilibrium, reaching a contradiction. Hence $\lim_{r\to 0}(n-k) = \infty$.

To show $\lim_{r\to 0} k = \infty$, we first establish a few additional asymptotic properties of the model.

Lemma A.1. (1) $rv_n \sim \pi_\infty - c\kappa$; (2) $v_{n+1} - v_n \sim c$; (3) $r\,(n-k) \sim 0$; (4) $rk \sim 0$.

Proof. (1) The fact that leader invests in state n but not in state $n+1$ implies

$$\frac{\pi_{n+2} - rv_{n+1}}{r+\kappa} = v_{n+2} - v_{n+1} \leq c \leq v_{n+1} - v_n = \frac{\pi_{n+1} - rv_n}{r+\kappa}$$

$$\Longrightarrow\ \pi_\infty - c\kappa = \lim_{r\to 0}(\pi_{n+2} - c\kappa) \geq \lim_{r\to 0} rv_n \geq \lim_{r\to 0}(\pi_{n+1} - c\kappa) = \pi_\infty - c\kappa,\ \text{Q.E.D.}$$

(2) The claim follows from the previous one: $v_{n+1} - v_n = \frac{\pi_{n+1}-rv_n}{r+\kappa} \sim \frac{\pi_\infty - rv_n}{\kappa} \sim c$.

(3) The previous claims show $rv_n \sim \pi_\infty - c\kappa$ and $\Delta v_n \sim c$. We apply Proposition A.1 to iterate backwards and obtain a lower bound for $(v_k - v_n)$:

$$\lim_{r\to\infty} r\left(v_k - v_n\right) \geq \lim_{r\to\infty} -\frac{r^2}{\kappa^2}\frac{rv_n - (\pi_\infty - c\eta)}{\left(1-\eta/\kappa\right)^4}\left(\eta/\kappa\right)^{n-k+1} \sim -\frac{r^2}{\kappa^2}\frac{c\left(\eta-\kappa\right)}{\left(1-\eta/\kappa\right)^4}\left(\eta/\kappa\right)^{n-k+1}$$

Since $\left|\lim_{r\to 0} r\left(v_k - v_n\right)\right| \leq \pi_\infty$, $\lim_{r\to 0} r^2\left(\eta/\kappa\right)^{n-k+1}$ must remain bounded, implying $r\left(n-k\right) \sim 0$.

(4) We apply Proposition A.1 to find a lower bound for $w_k - w_0$ (where $a \equiv \eta/\kappa > 1$):

$$\lim_{r\to 0} r\left(w_k - w_0\right) \geq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0 - (\pi_\infty - 2c\eta)}{a-1}\right)\frac{ra^k}{a-1} \geq \lim_{r\to 0}\left(\frac{2c\eta - \pi_\infty}{a-1}\right)\frac{ra^k}{a-1}.$$

Since $r\left(w_k - w_0\right)$ is bounded, it must be that ra^k is bounded; therefore $rk \sim 0$. QED.

Lemma A.2. $rv_{-k} \sim r\Delta v_{-k} \sim rv_{-n} \sim \Delta v_{-n} \sim 0$**.**

Proof. First, note that follower not investing in state $k+1$ implies $c \geq \Delta v_{-(k+1)}$. We apply Proposition A.1 to find an upper bound for $(v_{-n} - v_{-k})$ as a function of rv_{-k} and $\Delta v_{-(k+1)}$: $v_{-n} - v_{-k} \leq \lim_{r\to 0}\left(-\Delta v_{-(k+1)}\frac{\eta}{\eta-\kappa} + \left(n-k\right)\frac{rv_{-k}}{\eta-\kappa}\right)$, which implies $r\left(v_{-n} - v_{-k}\right) \sim 0$. Let $m \equiv \text{floor}(\frac{n+k}{2})$. That the follower does not invest in state m implies $c \geq \Delta v_{-m}$. Proposition A.1. provides a lower bound for $v_{-(n+1)} - v_{-n}$ as a function of rv_{-m} and Δv_{-m-1}: $\lim_{r\to 0}\left(v_{-(n+1)} - v_{-n}\right) \geq \lim_{r\to 0} -\Delta v_{-(m+1)}\left(\kappa/\eta\right)^{n-m} + \frac{rv_{-m} - \pi_{-m}}{\eta-\kappa} = \lim_{r\to 0}\frac{rv_{-m}}{\eta-\kappa}$, where the equality follows from $\lim_{r\to 0}\left(\kappa/\eta\right)^{n-m} = 0$ and $\lim_{r\to 0}\pi_{-m} \to 0$. Since the LHS is non-positive, it must be the case that $\lim_{r\to 0}\Delta v_{-n} = \lim_{r\to 0} rv_{-m} = 0$. But since $rv_{-n} \leq rv_{-m}$, it must be that $rv_{-n} \sim 0$, which, together with $rv_{-n} \sim rv_{-k}$, further implies $rv_{-k} \sim 0$. That $r\Delta v_{-k} \sim 0$ follows directly from the HJB equation for state k. QED.

We now prove $\lim_{r\to 0} k = \infty$**.** We show k bounded $\implies$ $rw_k \sim r\Delta w_k \sim 0$, and we look for a contradiction. First, we use the fact that $0 \leq \pi_{-s}$ for all $0 \leq s \leq k$ and apply Proposition A.1 (simplification 1a, substituting $u_s \equiv v_{-k+1}$, $u_{s+t} = v_0$, $t = k+1$, $\Delta u_s = \Delta v_{-k}$, $a = \frac{\eta}{\eta+\kappa}$, $b = \frac{r}{\eta+\kappa}$, $\delta = \frac{rv_{-(k+1)} - (-c\eta)}{\eta+\kappa}$) to find an asymptotic upper bound for rv_0:

$$\lim_{r\to 0} rv_0 = \lim_{r\to 0} r\left(v_0 - v_{-(k+1)}\right) \leq \lim_{r\to 0}\frac{r}{1-\kappa/\eta}\left(\Delta v_{-(k+1)} + k\frac{rv_{-(k+1)} + c\eta}{\eta}\right)$$

By Lemma A.1 part (4) and Lemma A.2, the RHS converges to 0, implying that $rv_0 \sim rw_0 \sim 0$. Further, using the HJB equation for state 0, we find that $\Delta w_0 \equiv w_1 - w_0 = \frac{rw_0+2c\eta-2\pi_0}{2\eta} \sim c - \pi_0/\eta$.

Lower and upper bounds for rw_k and $r\Delta w_k$ can be found, as functions of Δw_0 and rw_0, using Proposition A.1 (simplification 2(a), substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta+\kappa}{\eta}$, $b = \frac{r}{\eta}$, and $\delta = \frac{rw_0-(-2c\eta)}{\eta}$ for the upper bound, $\delta = \frac{rw_0-(\pi_\infty-2c\eta)}{\eta}$ for the lower bound):

$$\begin{aligned} \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0+2c\eta-\pi_\infty}{\kappa}\right)\frac{\eta}{\kappa}r\left(\frac{\eta+\kappa}{\eta}\right)^k &\leq \lim_{r\to 0}\left(rw_k - rw_0\right) \\ &\leq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0+2c\eta}{\kappa}\right)\frac{\eta}{\kappa}r\left(\frac{\eta+\kappa}{\eta}\right)^k \end{aligned} \tag{A.14}$$

$$\begin{aligned} \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0+2c\eta-\pi_\infty}{\kappa}\right)r\left(\frac{\eta+\kappa}{\eta}\right)^{k-1} &\leq \lim_{r\to 0}\left(r\Delta w_k\right) \\ &\leq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0+2c\eta}{\kappa}\right)r\left(\frac{\eta+\kappa}{\eta}\right)^{k-1}. \end{aligned} \tag{A.15}$$

If k is bounded, these inequalities imply $rw_k \sim r\Delta w_k \sim 0$.

Now suppose $rw_k \sim r\Delta w_k \sim 0$ and we look for a contradiction. Let $\hat{k} \equiv \max\{k, N\}$ where N is the smallest integer such that $\pi_N - \pi_0 > c\kappa$. That $|N-k|$ is finite and $rw_k \sim r\Delta w_k \sim 0$ jointly imply $rw_N \sim r\Delta w_N \sim 0$. Note that $\pi_{\hat{k}}$ is a lower bound for π_s for all $n \geq s \geq \hat{k}$; we apply Proposition A.1 (simplification 1, substituting $u_s \equiv w_{\hat{k}}$, $u_{s+t} = w_{n+1}$, $t = n+1-\hat{k}$, $\Delta u_s = \Delta w_{\hat{k}}$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_{\hat{k}}-\left(\pi_{\hat{k}}-c\eta\right)}{\eta}$) and obtain $\frac{rw_{\hat{k}}-\left(\pi_{\hat{k}}-c\eta\right)}{\eta-\kappa}$ as an asymptotic upper bound for $w_{n+1} - w_n$. Lemma A.1 part 2 further implies that

$$\lim_{r\to 0}\frac{rw_{\hat{k}} - (\pi_{\hat{k}} - c\eta)}{\eta-\kappa} \geq c \iff \lim_{r\to 0} rw_{\hat{k}} \geq \pi_{\hat{k}} - c\kappa > 0. \tag{A.16}$$

This contradicts the presumption that $rw_{\hat{k}} \sim 0$. QED.

Note that (A.14), (A.15), and the contradiction above jointly imply $\lim_{r\to 0} rw_k > 0$ and $\lim_{r\to 0} r\Delta w_k > 0$, and that $r\left(\frac{\eta+\kappa}{\eta}\right)^k$ converges to a positive constant. We summarize these findings into a Lemma.

Lemma A.3. $\lim_{r\to 0} r\Delta w_k > 0$, and $r\left(\frac{\eta+\kappa}{\eta}\right)^k$ converges to a positive constant as $r \to 0$.

Proof of Theorem 1. We show $\lim_{r\to 0} (\kappa/\eta)^{n-k} (1+\kappa/\eta)^k = 0$, which, based on Lemma 3, is a sufficient condition for $\mu^M \to 1$, $\mu^C \to 0$, and $g \to \kappa \cdot \ln \lambda$.

To proceed, we first find a lower bound for Δw_k by applying simplification 2 of Proposition A.1 (substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta+\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_0 - (\pi_\infty - 2c\eta)}{\eta}$):

$$\lim_{r\to 0} r\Delta w_k \geq \lim_{r\to 0} \left(\Delta w_0 + \frac{rw_0 - (\pi_\infty - 2c\eta)}{\kappa}\right) r\left(\frac{\eta+\kappa}{\eta}\right)^k. \tag{A.17}$$

Simplification 1 of Proposition A.1 provides asymptotic bounds for Δw_n (substituting $u_s = w_k$, $u_{s+t} = w_n$, $t = n-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; the upper bound is obtained using $\delta = \frac{rw_k - (\pi_k - c\eta)}{\eta}$ and the lower bound is obtained using $\delta = \frac{rw_k - (\pi_\infty - c\eta)}{\eta}$):

$$\lim_{r\to 0}\left[\Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi_k}{\eta-\kappa}\right] \geq \lim_{r\to 0} \Delta w_n$$

$$\lim_{r\to 0} \Delta w_n \geq \lim_{r\to 0}\left[\Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi_\infty}{\eta-\kappa}\right].$$

Since $\lim_{r\to 0} \pi_k = \pi_\infty$, the lower and upper bounds coincide asymptotically. Furthermore, Lemma A.1 shows $\Delta w_n \sim c$; hence,

$$c \sim \Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi_\infty}{\eta-\kappa}. \tag{A.18}$$

Next, we apply simplification 1(b) of Proposition A.1 to obtain (substituting $u_s \equiv w_k$, $u_{s+t} = w_n$, $t = n-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; the simplification applies because $\lim_{r\to 0} r\Delta w_k > 0$, as stated in Lemma A.3): $r(w_n - w_k) \sim \frac{r\Delta w_k}{(\eta-\kappa)/\eta}$. Part 1 of Lemma A.1 further implies

$$\pi_\infty - c\kappa - rw_k \sim \frac{r\Delta w_k}{(\eta-\kappa)/\eta}. \tag{A.19}$$

Substituting the asymptotic equivalence (A.19) into (A.18), we obtain

$$c \sim c + \Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) - \frac{r\eta\Delta w_k}{(\eta-\kappa)^2}$$

$$\implies 0 \sim \Delta w_k \left(\kappa/\eta\right)^{n-k}.$$

Further substitute into inequality (A.17),

$$0 \geq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0 - (\pi_\infty - 2c\eta)}{\kappa}\right)\left(\frac{\eta+\kappa}{\eta}\right)^k \left(\kappa/\eta\right)^{n-k}$$

Given $\Delta w_0 \geq 0$, $rw_0 \geq 0$, and $2c\eta - \pi_\infty > 0$, the inequality can hold if and only if $\lim_{r\to 0}\left(\frac{\eta+\kappa}{\eta}\right)^k \left(\kappa/\eta\right)^{n-k} = 0$, as desired. All other claims in Theorem 1 follows directly. QED.

Finally, the next result characterizes the relative rate of divergence between $(n-k)$ and k, as well as the rate of convergence of μ^M.

Lemma A.4. 1) $\lim_{r\to 0}\frac{n-k}{k} = \frac{2\ln(1+\kappa/\eta)}{\ln \eta/\kappa}$; 2) $\lim_{r\to 0}\frac{1-\mu^M}{r}$ converges to a positive constant.

Proof of Lemma A.4. We first prove $\frac{n+k}{k} \sim \frac{2\ln(1+\alpha)}{-\ln \alpha}$. Note Lemmas A.1 and A.2 jointly imply $\frac{rw_{n+1}-(\pi_\infty - c\eta)}{\eta-\kappa} \sim c \sim \Delta w_n$. We apply Proposition A.1 simplification 2(b) to find $\lim_{r\to 0} rw_k$. We substitute $u_s = w_{n+1}$, $u_{s+t} = w_k$, $\Delta u_s = w_n - w_{n+1} = -\Delta w_n$, $a = \frac{\eta}{\kappa}$, $b = \frac{r}{\kappa}$; the upper bound is obtained using $\delta = \frac{rw_{n+1}-(\pi_k - c\eta)}{\kappa}$ and the lower bound is obtained using $\delta = \frac{rw_{n+1}-(\pi_\infty - c\eta)}{\kappa}$, and that the lower and upper bounds coincide as $r \to 0$. Simplification 2(b) applies because $\Delta u_s + \frac{\delta}{a-1} \sim -c + \frac{rw_{n+1}-(\pi_\infty - c\eta)}{\kappa(\eta/\kappa - 1)} \sim 0$. Proposition A.1 implies

$$w_k - w_{n+1} \sim -\frac{r}{\kappa\left(\eta/\kappa - 1\right)^4}\frac{c\left(\eta-\kappa\right)}{\kappa}\left(\eta/\kappa\right)^{n+1-k}$$

$$\implies r\left(w_{n+1} - w_k\right) \sim \frac{c\left(\eta-\kappa\right)}{\kappa^2\left(\eta/\kappa - 1\right)^4} r^2 \left(\eta/\kappa\right)^{n+1-k}$$

$$\text{substitute into (A.19)} \implies r\Delta w_k \sim \varphi_1 \cdot r^2 \left(\eta/\kappa\right)^{n-k} \quad \text{for some constant } \varphi_1 > 0.$$

We denote $a = \Phi\left(f\left(r\right)\right)$ if $a/f\left(r\right)$ converges to a positive constant as $r \to 0$. By Lemma A.3, $\lim_{r\to 0} r\Delta w_k > 0$, hence $\left(\kappa/\eta\right)^{n-k} = \Phi\left(r^2\right)$. Lemma A.3 also states that $\left(1+\kappa/\eta\right)^{-k} = \Phi\left(r\right)$; hence $\left(\eta/\kappa\right)^{n-k} \sim \varphi_2\left(1+\kappa/\eta\right)^{2k}$ for some constant $\varphi_2 > 0$, im-

plying

$$(n-k)\ln(\eta/\kappa) \sim \ln\varphi_2 + 2k\ln\left(\frac{\eta+\kappa}{\eta}\right)$$

$$\implies \frac{n-k}{2k} \sim \frac{2\ln(1+\kappa/\eta)}{\ln\eta/\kappa}, \quad \text{as desired.}$$

We now prove $1-\mu^M = \Phi(r)$. By Lemma 3 and denoting $\alpha \equiv \kappa/\eta$,

$$1-\mu^M = \frac{\alpha^{n-k}\left((1+\alpha)^k - 1\right) + \alpha^{n-k+1}(1+\alpha)^k/2}{\frac{1-\alpha^{n-k+1}}{1-\alpha} + \alpha^{n-k}\left((1+\alpha)^k - 1\right) + \alpha^{n-k+1}(1+\alpha)^k/2}.$$

Hence $\left(1-\mu^M\right) \sim (\kappa/\eta)^{n-k}(1+\kappa/\eta)^k$. But we have established above that $(\kappa/\eta)^{n-k} = \Phi(r^2)$ and $(1+\kappa/\eta)^{-k} = \Phi(r)$; jointly, these asymptotic relationships imply $1-\mu^M = \Phi(r)$, as desired.

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